

Link Budget Analysis: Digital Modulation, **Part 3**

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Presentation Content

Link Budget Analysis: Digital Modulation, **Part 3**

1. PSK: Phase Shift Keying.
 - A. Basis of Operation.
 - B. Signal Constellation Diagrams.
2. Binary Phase Shift Keying: BPSK
 - A. Basis of Operation.
 - B. Typical BPSK Modulated Signal.
 - C. Implementation of BPSK.
 - D. Power Spectral Density of BPSK.
 - E. Differential Phase Shift Keying.
3. Quadrature Phase Shift Keying: QPSK
 - A. Basis of Operation.
 - B. Power Spectral Density of QPSK.
 - C. Offset-QPSK & $\pi/4$ -Shifted QPSK.
 - D. QPSK Modulator & Demodulator.
4. 8-QPSK operation & Tx modulator.
5. Power Spectral Density of M-PSK.
6. BER for Multi-Level PSK.
7. QAM: Quadrature Amplitude Modulation
 - A. Overview & Basis of Operation.
 - B. BER for Multi-Level QAM.
 - C. QAM in Digital Cable Services.
 - D. 4-QAM & 8-QAM: Overview.
 - E. 8-QAM Transmit Modulator.
 - F. 16-QAM: Constellation Diagrams.
 - G. 16-QAM Transmit Modulator.
 - H. Power Spectral Density of M-QAM.
8. Comparison: M-PSK versus M-QAM.
9. Practical Applications: PSK & QAM.
10. MER: Modulation Error Ratio.
11. Summary: Digital Modulation, Part 3.

Refer to background material in Atlanta RF's presentations titled:

1. 'Link Budget – Getting Started' and
2. 'Link Budget: Digital Modulation Part 1' (Overview & M-ASK)
3. 'Link Budget: Digital Modulation Part 2' (M-FSK)

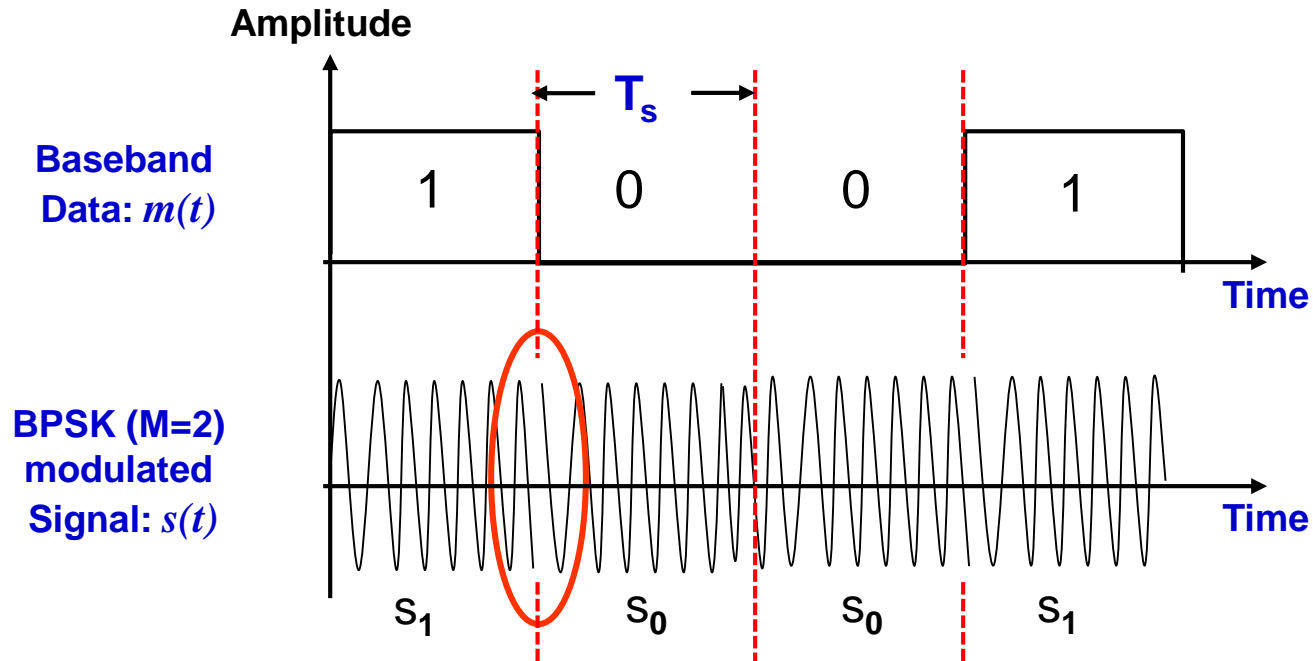
which can be downloaded from our website: www.AtlantaRF.com.

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PSK: Phase Shift Keying

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where $s(t)_0 = -A\cos(\omega_c t)$ and $s(t)_1 = A\cos(\omega_c t)$

BPSK can be expanded to a M-ary modulation scheme, employing multiple phases and multiple amplitudes as different states.

PSK: Phase Shift Keying

Basis of operation

1. When the baseband signal modulates the phase of the carrier signal, the process is called “[phase modulation](#)”. For digital baseband signals, it is called “[Phase Shift Keying](#)”: PSK .
2. Phase-Shift Keying (PSK) is a form of [digital modulation](#) that represents [digital data](#) solely through discrete variations in the [phase](#) of a [carrier's signal](#).
3. In PSK, the phase of the carrier signal is changed abruptly between 2 (for BPSK) or more (for MPSK) phase states by the baseband digital message signal: $m(t)$, at the beginning of each signal interval: T_s , to represent binary bits ‘0’ or ‘1’ (or a pair or trio of bits). The carrier signal's amplitude and frequency remain constant.

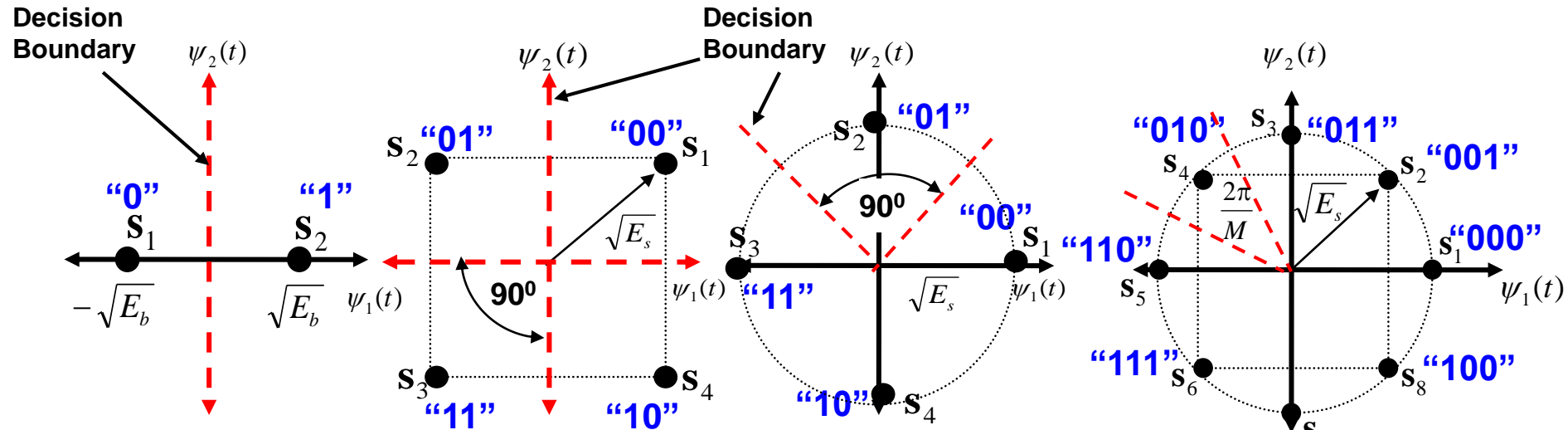
$$S_i(t) = \sqrt{\frac{2E_s}{T_s}} \cos(2\pi f_c t + \theta_i), 0 \leq t \leq T_s ; i = 1 \text{ to } M$$

$$\text{where : } \theta_i = \frac{2\pi i}{M} ; E_s = (\log_2 M) E_b ; T_s = (\log_2 M) T_b ; f_c = n_c / T_s$$

4. Multi-Level PSK (M-PSK) produces ‘M’ phase states around a constellation diagram circle, with each phase state separated by $2\pi/M$ radians.
5. PSK is not susceptible to the noise degradations that affects ASK, or to the bandwidth limitations of FSK. PSK is, however, limited by the ability of the equipment to distinguish small differences in phase. This factor limits its potential transmission bit rate.

Multi-Level PSK: Signal Constellation Diagrams

Two-dimensional linear modulation



BPSK (M = 2)

$k = 1$ -bit/state

$M = 2^1 = 2$ states

$\Delta\theta = 180^\circ$ step

$d = 2\sqrt{E_s} = 2\sqrt{E_b}$

QPSK (M = 4)

$k = 2$ -bits/state

$M = 2^2 = 4$ states

$\Delta\theta = 90^\circ$ steps

$d_{\min} = \sqrt{2E_s} = 2\sqrt{E_b}$

QPSK also

$k = 2$ -bits/state

$M = 2^2 = 4$

$\Delta\theta = 90^\circ$ steps

$d_{\min} = \sqrt{2E_s} = 2\sqrt{E_b}$

8-PSK (M = 8)

$k = 3$ -bits/state

$M = 2^3 = 8$ states

$\Delta\theta = 45^\circ$ steps

$d_{\min} = 0.7654\sqrt{E_s}$

where the minimum distance between phase states is: $d_{\min} = 2\sqrt{E_s} \sin\left(\frac{\pi}{M}\right)$

and the Energy per symbol is: $E_s = E_b \log_2 M = kE_b$

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BPSK: Binary Phase Shift Keying

Basis of operation

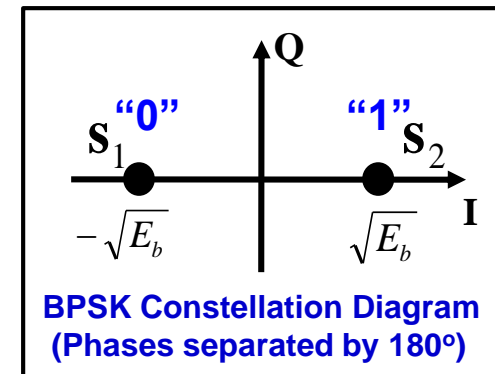
1. **Binary PSK (BPSK)** is a 2-level modulation ($M = 2$) technique, where the digital message state for a binary '0' and a binary '1' are represented by two different phase states in the carrier's signal: $\theta = 0^\circ$ for binary '0' and $\theta = 180^\circ$ for binary '1' (antipodal signals). During T_b , the duration of one information bit during the signaling interval, the two modulated carrier signals can be expressed as:

$$s_1(t) = A_c \cos(2\pi f_c t) = \sqrt{E_b} \phi_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t), \text{ binary '0'}$$

$$s_2(t) = A_c \cos(2\pi f_c t + \pi) = \sqrt{E_b} \phi_2(t) = -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t), \text{ binary '1'}$$

where the carrier frequency: $f_c = n_c/T_b$ for fixed integer n_c .

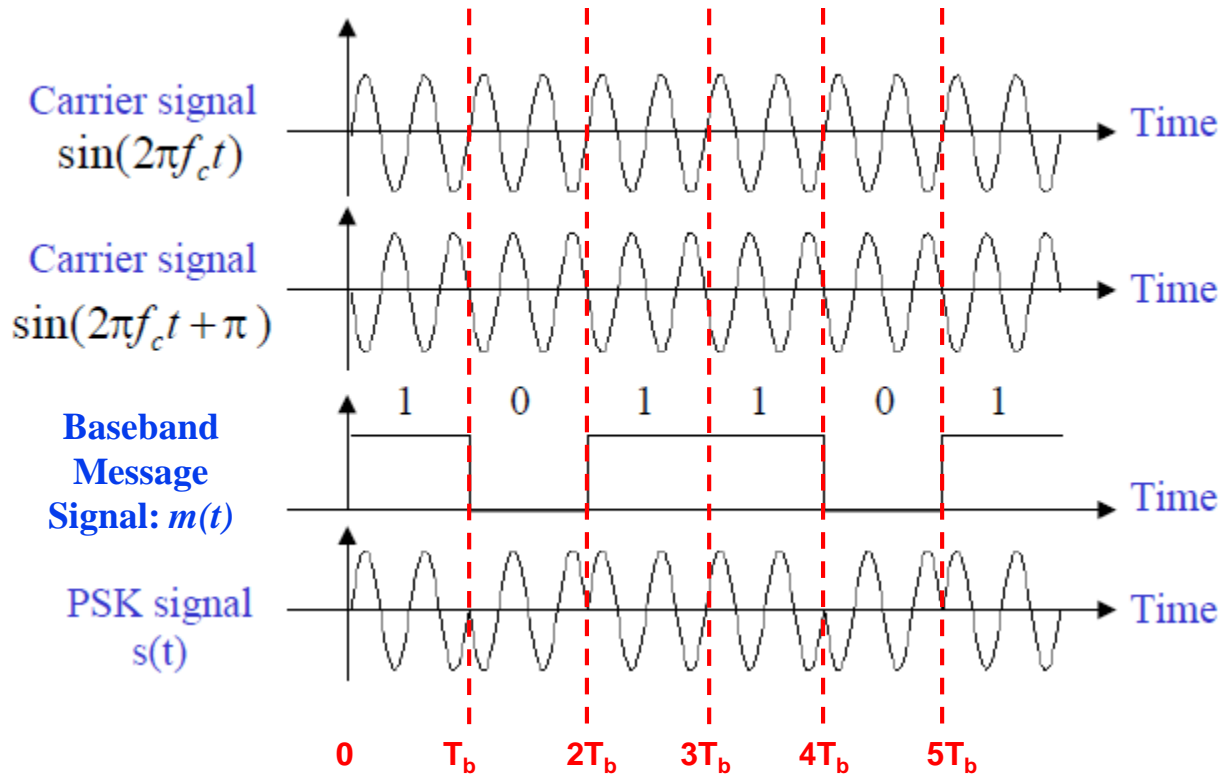
2. Carrier phases are 180° apart. Constant amplitude envelop.
3. All the information-bearing data is in the signal's phase.
4. Simple to implement, but inefficient use of bandwidth.
5. Binary PSK is a very robust modulation technique that is used extensively in satellite digital communications.



Typical Binary PSK Modulated Signal

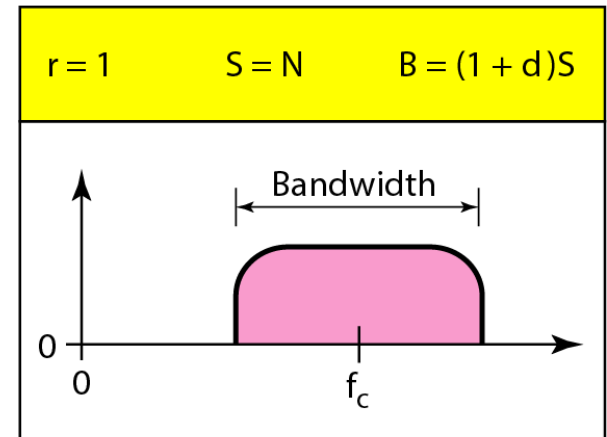
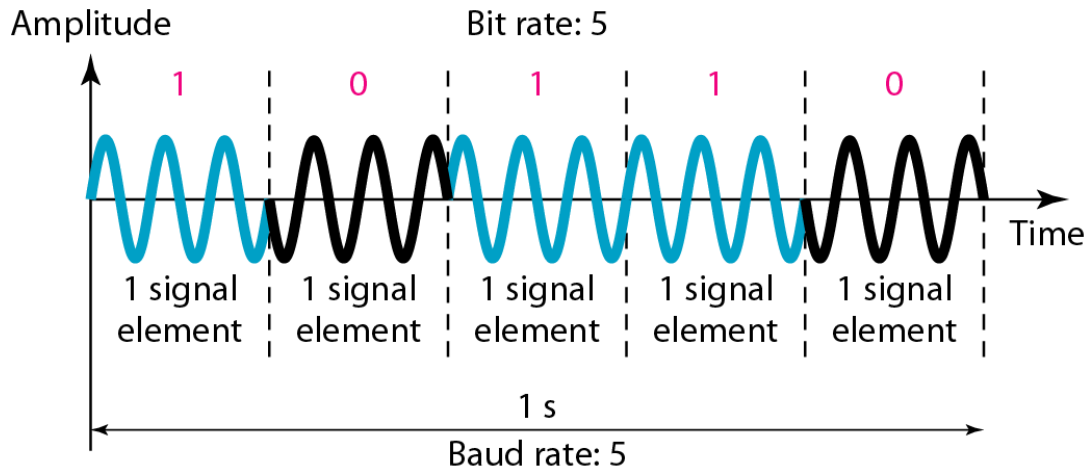
BPSK waveforms shown in the time domain

Binary bit '1' and '0' are represented by one carrier frequency with two different phase states: 0° and 180°

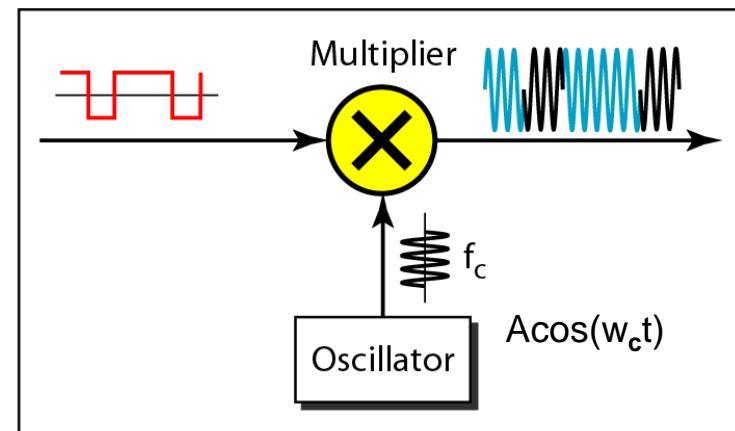
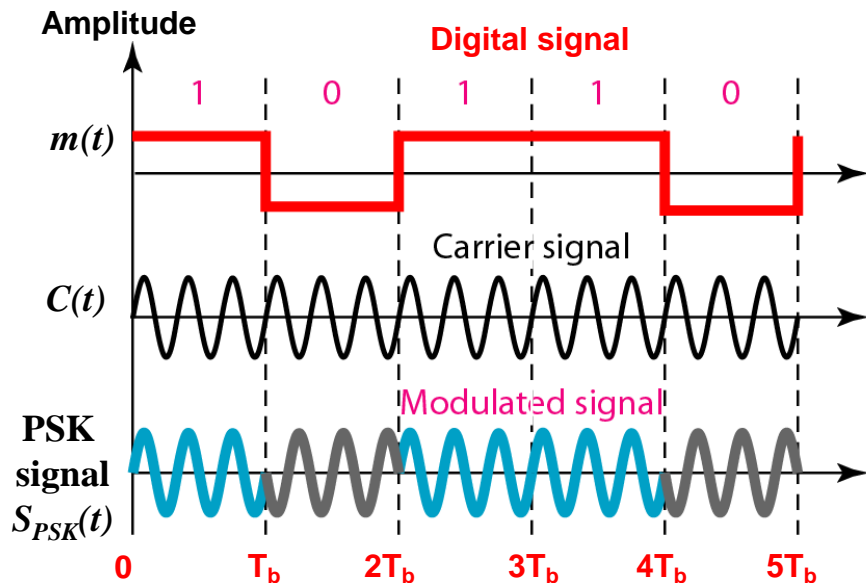


BPSK: Binary Phase Shift Keying

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Implementation of BPSK:

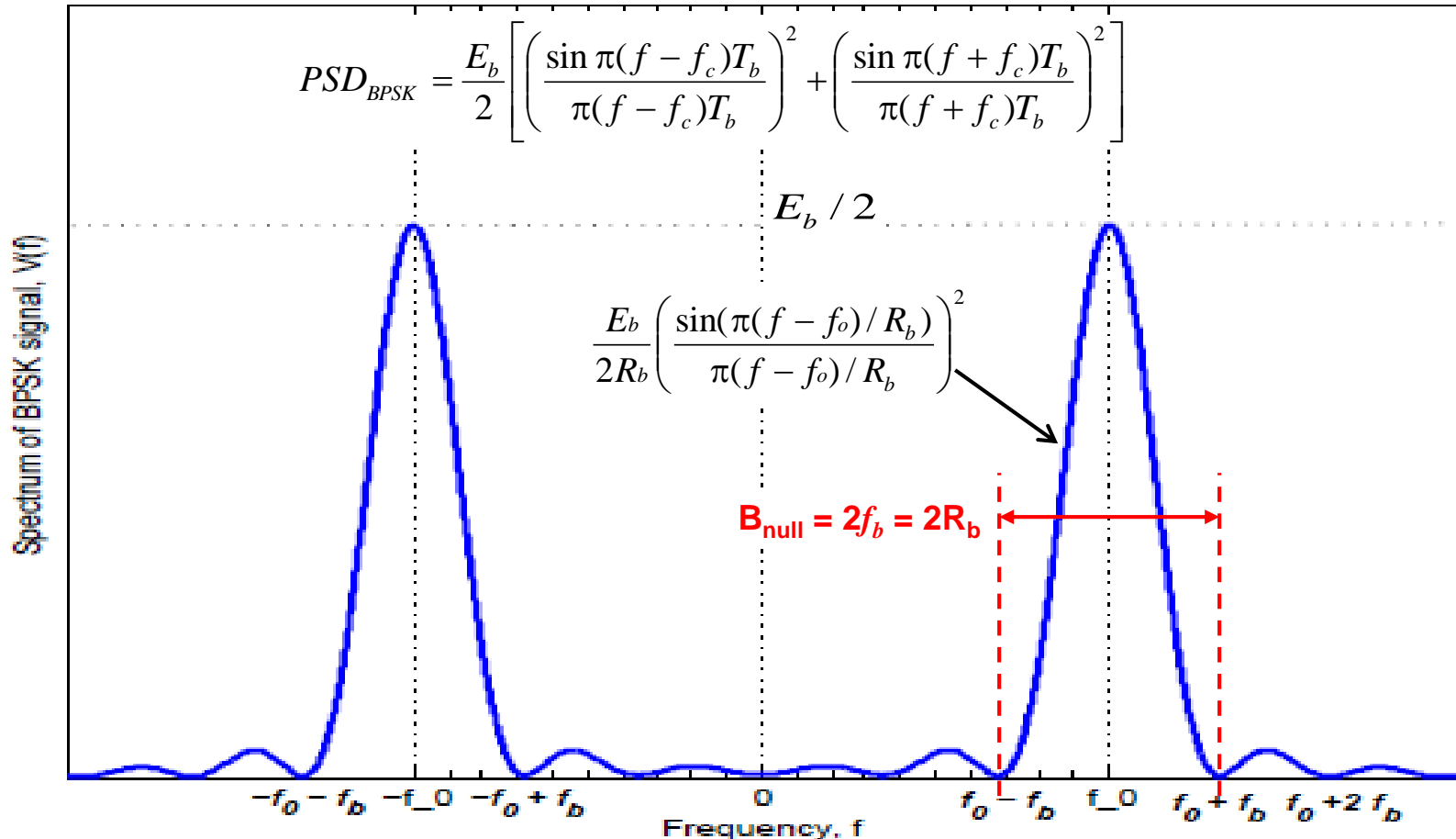


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Bandpass Power Spectral Density of Binary PSK

Assumes baseband rectangular pulses: 



- Null-to-Null RF bandwidth: $B_{BPSK} = (f_o + f_b) - (f_o - f_b) = 2f_b = 2R_b = 2/T_b$.
- 90% power RF bandwidth: $B_{90\%} = 1.6f_b = 1.6R_b$ for rectangular pulses.
- 99% power RF bandwidth: $B_{99\%} = 20f_b = 20R_b$.

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DPSK: Differential Phase Shift Keying

Basis of Operation

1. For DPSK, the term 'differential' is used because the phase shift is with reference to the previous bit. However, the frequency of the reference signals must be the same as the received signal's frequency.

2. The information-bearing data is carried by the phase difference between the current symbol and the previous symbol:

$$\theta_{k+1} - \theta_k = \phi_i = 2\pi i / M$$

3. In the transmitter, each symbol is modulated relative to the phase of the immediately preceding signal element transmitted:

A. Binary '0': Signal burst with the same phase as the previous signal burst.

B. Binary '1': Signal burst of opposite phase to the previous signal burst.

4. DPSK does not require an accurate receive Local Oscillator that is matched with the transmitter for the phase information, but obviously depends on the preceding phase (information bit) being received correctly.

5. Power Spectral Density (PSD) for DPSK is same as PSD for BPSK; DPSK's Null-to-Null RF transmission bandwidth: $B_{\text{null}} = 2f_b$.

6. Probability of bit error for Differential PSK is: $P_{be,DPSK} = \frac{1}{2} \exp\left(-\frac{E_b}{N_0}\right)$

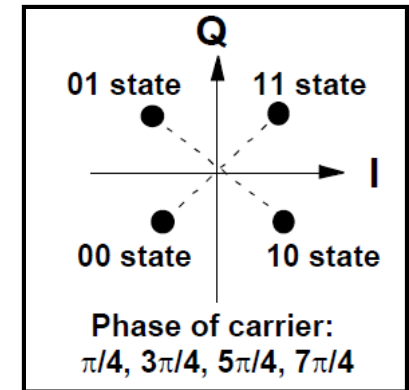
Sample index, k	0	1	2	3	4	5	6	7	8	9	10
Information message, $m(k)$		1	1	0	1	0	1	1	0	0	1
Differentially encoded message (first bit arbitrary), $c(k)$	1	1	1	0	0	1	1	1	0	1	1
Corresponding phase shift, $\theta(k)$	π	π	π	0	0	π	π	π	0	π	π

QPSK: Quadrature Phase Shift Keying

Basis of Operation

1. Quadrature PSK (QPSK) is a 4-level modulation technique ($M = 2^2 = 4$ states), where every two binary bits are combined and mapped (i.e.: encoded) to one of 4 phase states of the carrier signal. These 4 phases are located in 90° phase increments. If T_s indicates the duration of one information bit during the signaling interval, the four modulated carrier signals can be expressed as:

$$S_{QPSK}(t) = \begin{cases} S_1(t) = A_c \cos(2\pi f_c t + \theta_1) = \sqrt{\frac{2E_s}{T_s}} \cos(2\pi f_c + \pi/4)t & , \text{ for binary } 11 \\ S_2(t) = A_c \cos(2\pi f_c t + \theta_2) = \sqrt{\frac{2E_s}{T_s}} \cos(2\pi f_c + 3\pi/4)t & , \text{ for binary } 01 \\ S_3(t) = A_c \cos(2\pi f_c t + \theta_3) = \sqrt{\frac{2E_s}{T_s}} \cos(2\pi f_c + 5\pi/4)t & , \text{ for binary } 00 \\ S_4(t) = A_c \cos(2\pi f_c t + \theta_4) = \sqrt{\frac{2E_s}{T_s}} \cos(2\pi f_c + 7\pi/4)t & , \text{ for binary } 10 \end{cases}$$



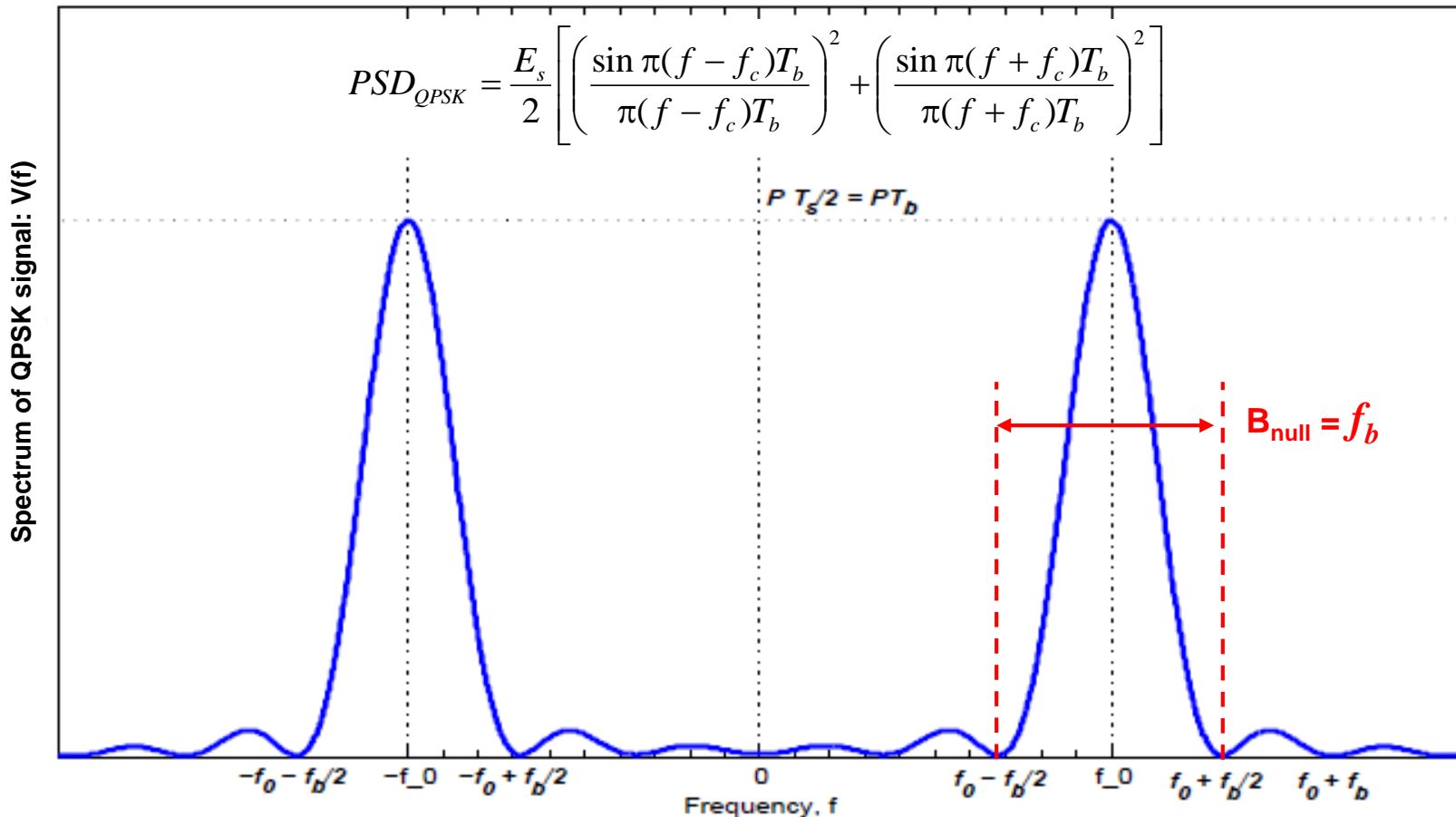
Phase constellation diagram for QPSK

where: $f_c = n_c/T_b$ for fixed integer n_c ; $E_s = E_b \log_2 M = 2E_b$ and $T_s = T_b \log_2 M = 2T_b$.

2. One of 4 possible waveforms is transmitted during each signaling interval: T_s .
3. The signaling rate (i.e., the baud rate) is $R_s = 1/T_s = 1/(2T_b) = R_b/2$ (symbols/sec),
4. QPSK can be interpreted as two independent BPSK systems in phase quadrature (one on the in-phase I-channel and one on the quadrature-phase Q-channel), and thus the same performance but twice the bandwidth efficiency of BPSK.

Bandpass Power Spectral Density of QPSK

Assumes baseband rectangular pulses: 



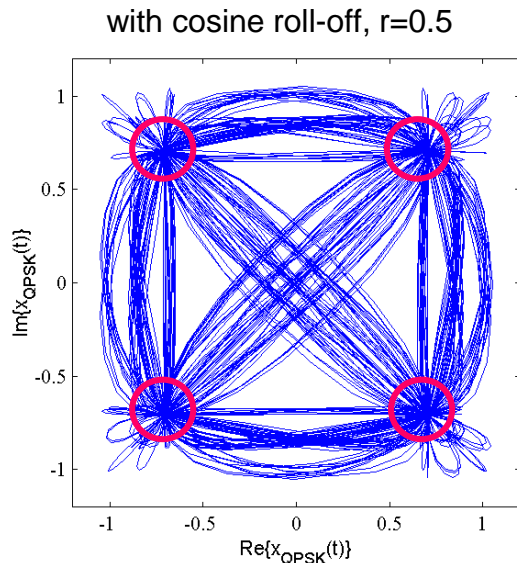
- $E_b = P T_s/2 = P T_b$, where: $T_s = T_b \log_2(2^2) = 2T_b$ ($M = 2^2 = 4$ states).
- Null-to-Null RF transmission bandwidth: $B_{\text{null}} = (f_0 + f_b/2) - (f_0 - f_b/2) = f_b = R_b$.
- Bandwidth with 90% of signal power: $B_{90\%} = 0.75f_b = 0.75R_b$.

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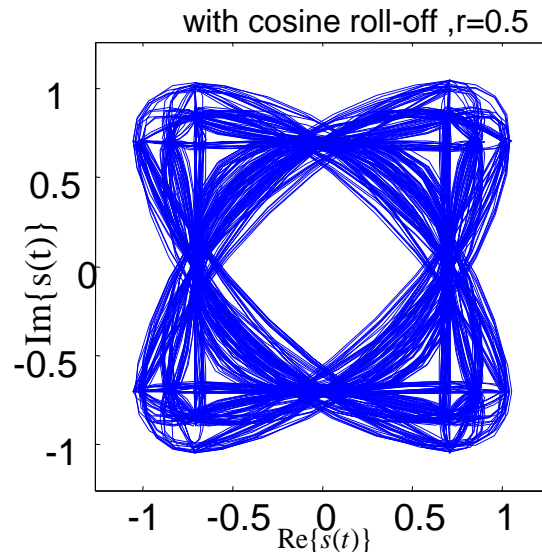
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Types of Quadrature Phase Shift Keying (QPSK)

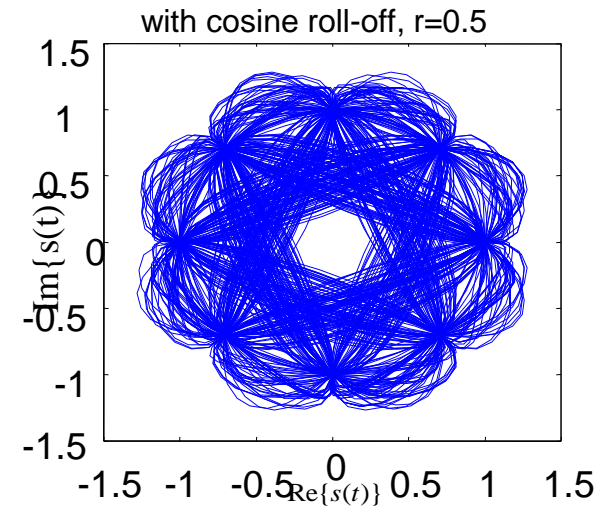
When viewed by Signal-Space Phase Constellation Diagram



Conventional QPSK



Offset QPSK



$\pi/4$ QPSK

1. **Conventional QPSK** has phase transitions through zero (i.e. 180° phase transitions). As such, highly linear amplifiers are required.
2. In **Offset QPSK**, the phase transitions on the in-phase I-channel and quadrature-phase Q-channel are staggered. Phase transitions are therefore limited to 90° .
3. In **$\pi/4$ QPSK**, the set of phase constellation points are toggled for each symbol, so transitions through zero cannot occur. This modulation scheme produces the lowest envelope variations.
4. **All QPSK modulation schemes require linear power amplifiers .**

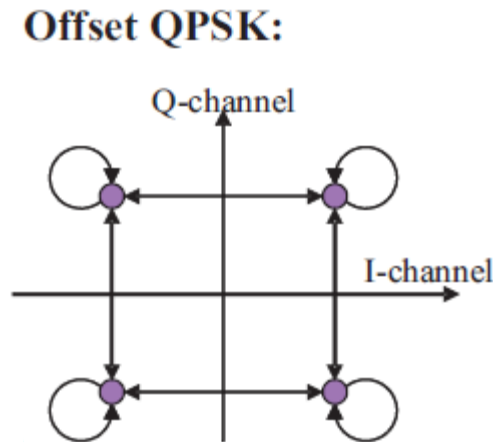
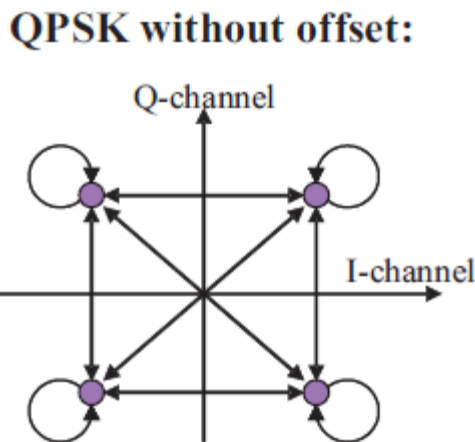
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QPSK: Offset QPSK Modulation

Basis of Operation: Variant of QPSK

1. Offset-QPSK is 4-level ($M = 2^2 = 4$) PSK, where the data transition times for the in-phase bit stream and quadrature-phase bit stream are offset by $\frac{1}{2}$ symbol period (or: one bit interval T_b).
2. OQPSK has phase transitions between every half-bit time that never exceeds 90 degrees.
3. Results in much less amplitude variation of the bandwidth-limited carrier signal.
4. Since staggering the bit streams does not change the orthogonality of the carriers, Offset-QPSK has the same theoretical bit error performance (BER) as BPSK and QPSK.
5. When amplified, QPSK results in significant band-width expansion, whereas OQPSK has much less bandwidth expansion, especially if the channel has non-linear components.



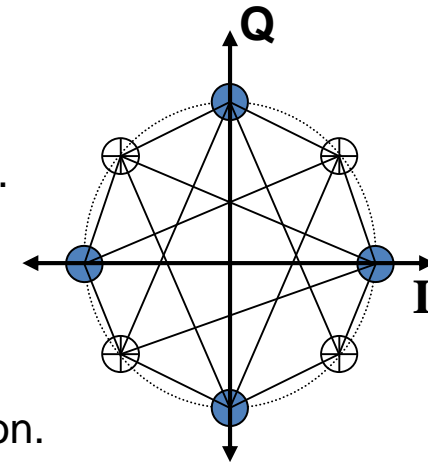
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$\pi/4$ -Shifted QPSK Modulation

Basis of Operation: Variant of QPSK

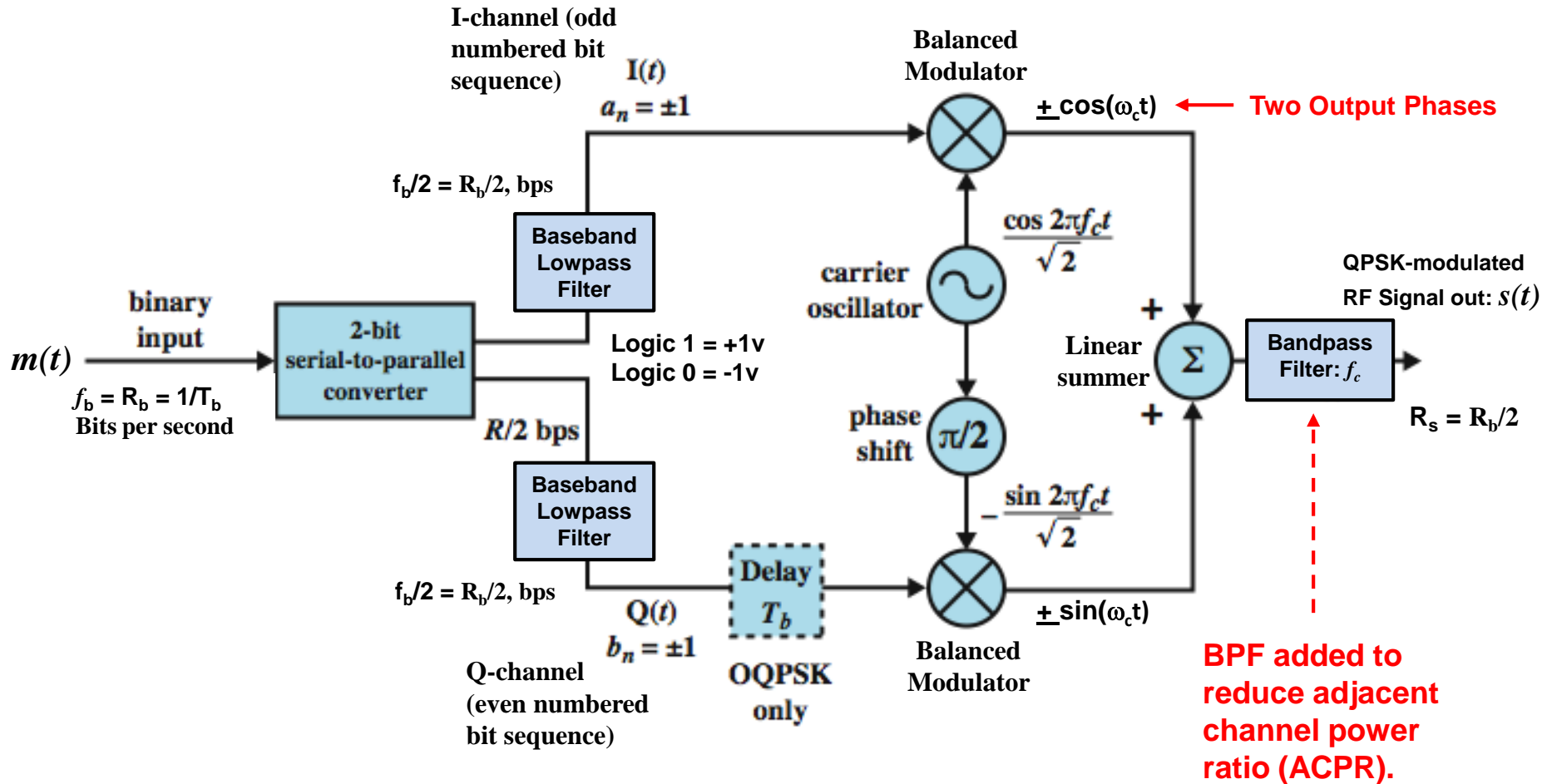
1. $\pi/4$ -Shifted QPSK is a variant of 4-phase PSK (QPSK):
 - A. Is obtained by introducing an additional $\pi/4$ phase shift in the carrier phase in each symbol interval. $\pi/4$ QPSK signal is generated by alternating between two QPSK constellations that are rotated by $\pi/4$ with respect to each other.
 - B. Restricts carrier phase transition to odd multiples of $\pi/4$ (45°).
 - C. Signaling elements selected from two QPSK constellations of four signal points each, with each constellation shifted by $\pi/4$.
 - D. Maximum phase change is limited to $\pm 135^\circ$ vs. 180° for QPSK, thus maintaining constant envelope (i.e.: Amplitude of a QPSK signal is not constant for the short time interval during 180° phase changes).
 - E. Two data bits per symbol. Twice as efficient as GMSK modulation.
 - F. Always a phase change between symbols, leading to self-clocking.
2. Popular in 2nd Generation Mobile Communication Systems:
 - A. North American Digital Cellular (IS-54): 1.62 bps/Hz (= 48kbps/30kHz).
 - B. Japanese Digital Cellular System: 1.68 bps/Hz.
 - C. European TETRA System: 1.44 bps/Hz.
 - D. Japanese Personal Handy Phone System (PHP).
 - E. Adopted by PACS (Personal Access Communication System).



Constellation Diagram
of $\pi/4$ QPSK

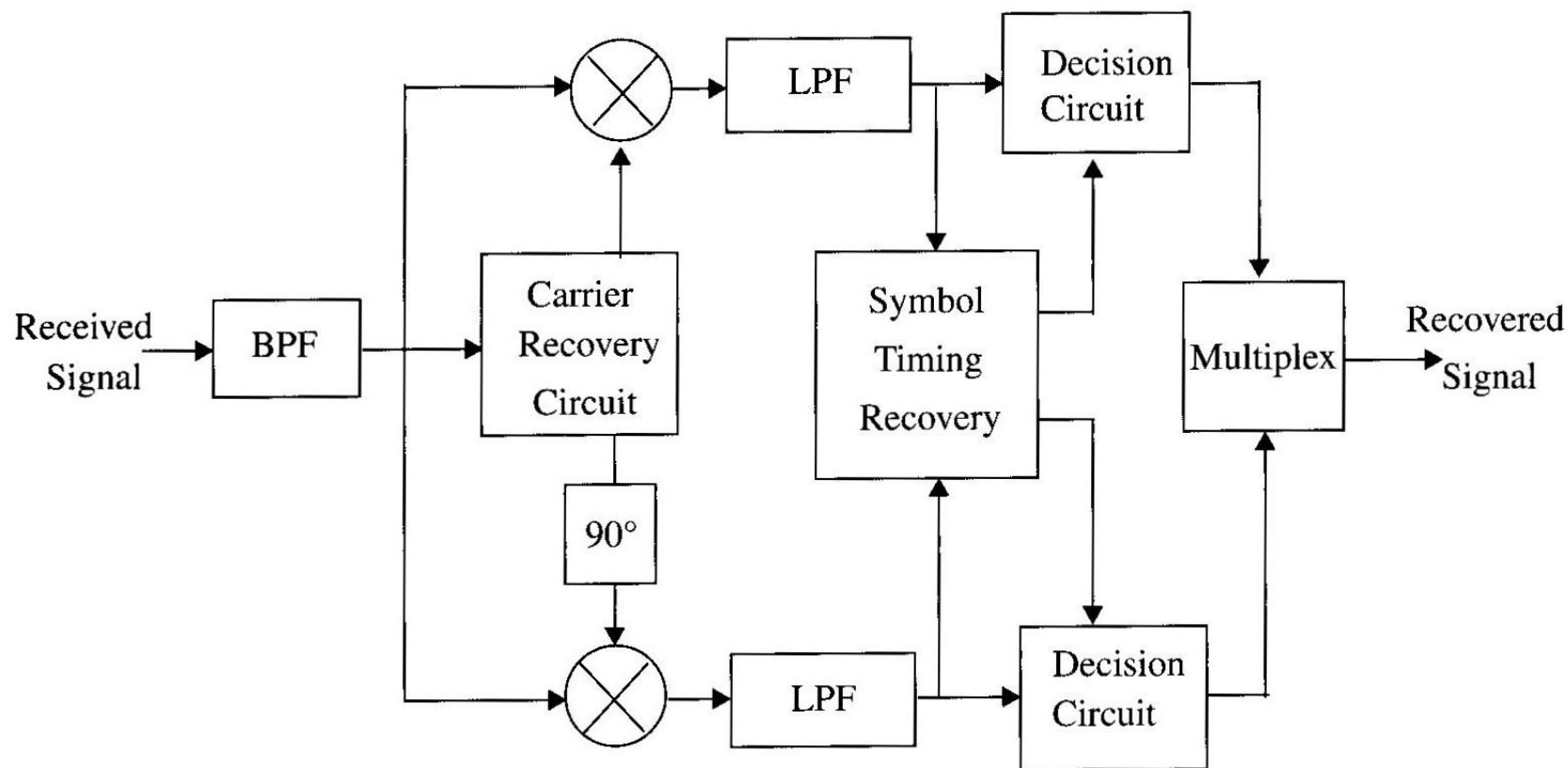
Transmit Modulator

For: Quadrature Phase Shift Keying & Offset-QPSK modulation



Receive Demodulator

For: Quadrature Phase Shift Keying (QPSK) demodulation



8-PSK: 8-level Phase Shift Keying Modulation

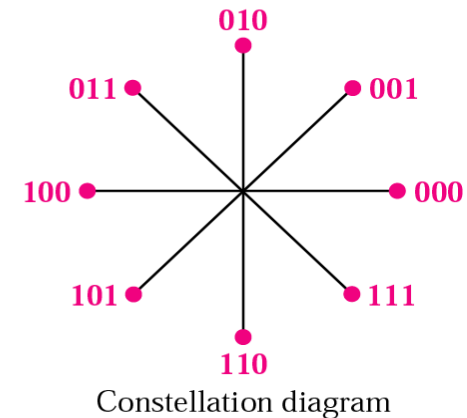
Basis of Operation

1. **8-PSK** is an 8-level modulation technique ($M = 2^3 = 8$ states) that encodes $k = 3$ bits (a 'tribit') of digital information into each of 8 possible phase states in the carrier's signal, and which are located in phase increments = 45° around the constellation phase diagram.

- A. 8-PSK is 3 times as efficient as BPSK.
- B. Minimum bandwidth required for PSK transmission is the same as that required for ASK transmission.
- C. Maximum bit rate in PSK transmission is much greater than that of ASK.
- D. While the maximum baud rates of ASK and PSK are the same for a given bandwidth, PSK bit rates using the bandwidth can be 2 or more times greater.

Tribit	Phase
000	0
001	45
010	90
011	135
100	180
101	225
110	270
111	315

Tribits
(3 bits)



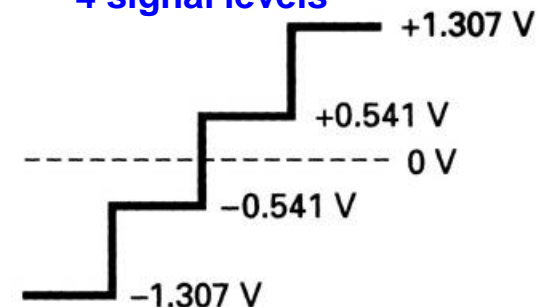
I-Channel Truth Table

I	C	Output
0	0	-0.541 V
0	1	-1.307 V
1	0	+0.541 V
1	1	+1.307 V

Q-Channel Truth Table

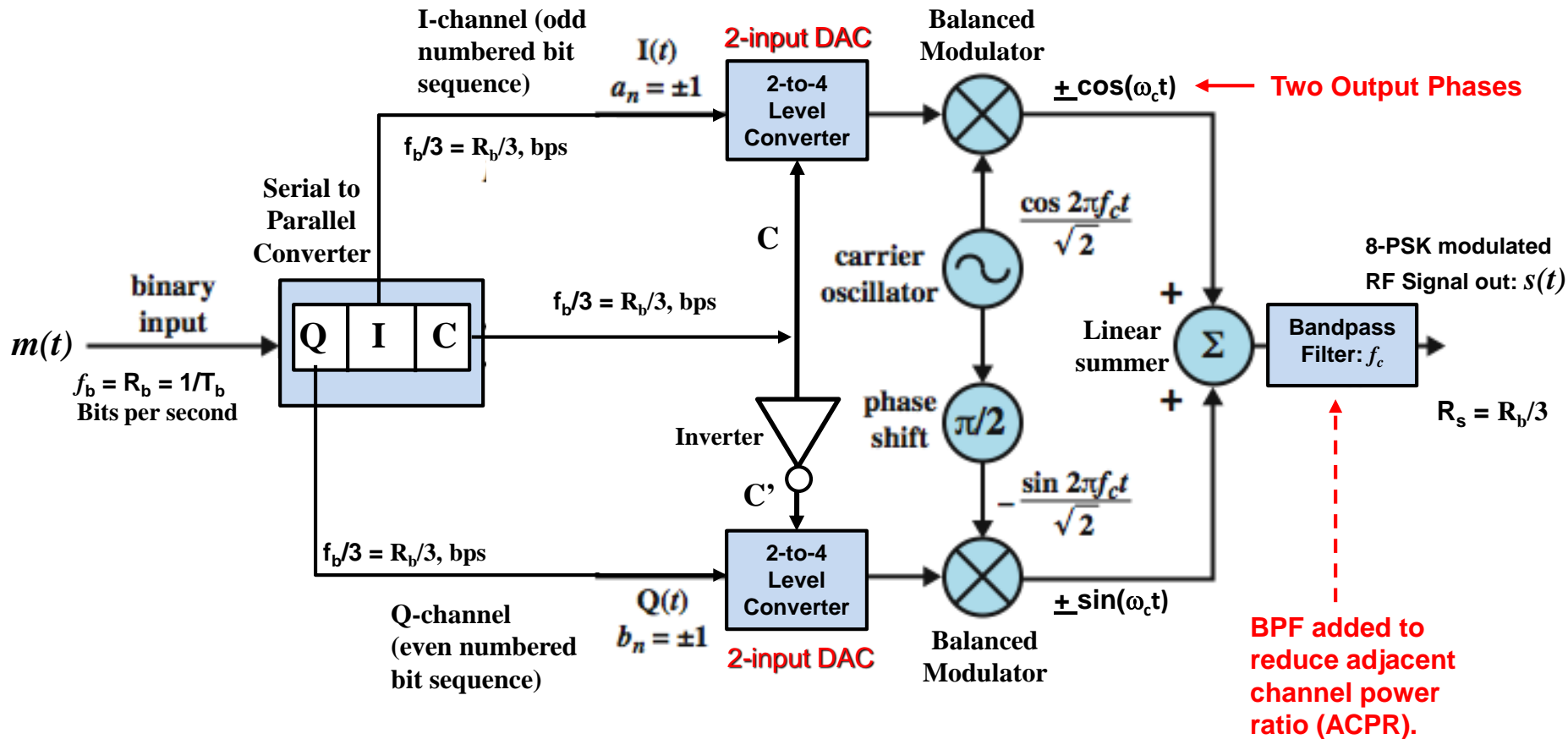
Q	\bar{C}	Output
0	1	-1.307 V
0	0	-0.541 V
1	1	+1.307 V
1	0	+0.541 V

4 signal levels



Transmit Modulator

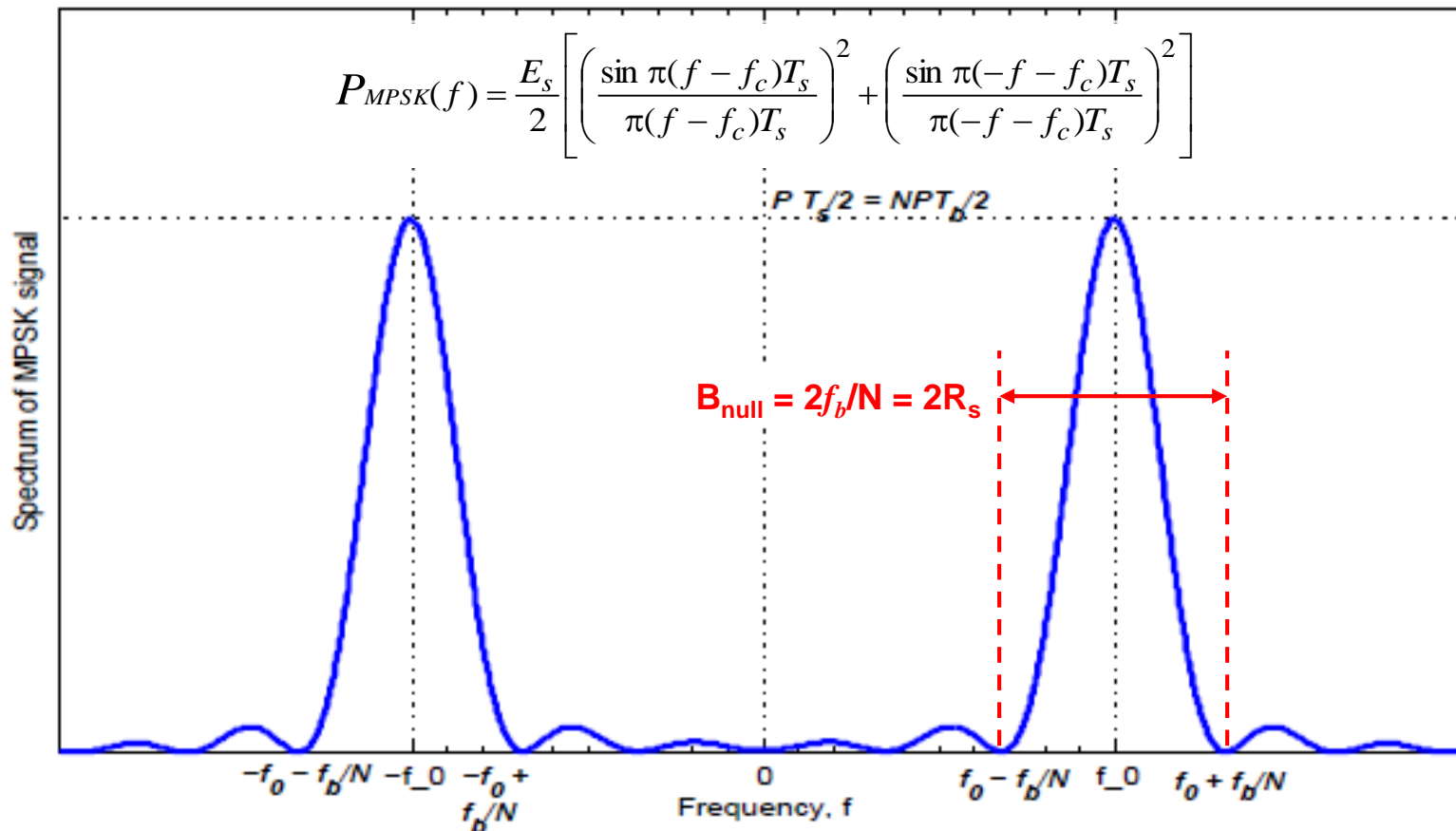
For: 8-Level Phase Shift Keying (8-PSK) modulation



- I, Q determines polarity; Binary '0' = Negative sign ("-").
Binary '1' = Positive sign ("+").
- C, \bar{C} determines the voltage level; Binary '1' = 1.307volts,
Binary '0' = 0.541volts.
- 2 levels + 2 polarities give 4 conditions in each (I & Q) channel.

Bandpass Power Spectral Density of **MPSK** signal

Assumes baseband rectangular pulses: 



- $E_s = P T_s$, watt-second; where: $T_s = T_b \log_2 M$ and $M = 2^N$ phase states.
- Null-to-Null RF transmission bandwidth: $B_{mpsk} = (f_0 + f_b/N) - (f_0 - f_b/N) = 2f_b/N = 2R_s$.
- Bandwidth Efficiency: $\eta_{BW} = R_b/B_{null} = (\log_2 M)/(1+\alpha)$, bits/sec/Hertz.

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Probability of Bit Error (BER) for M_{ary} PSK Modulation In an Additive White Gaussian Noise (AWGN) channel

1. The average symbol error probability of a coherent M -ary PSK system is:

$$P_{se,MPSK} \cong 2Q\left[\sqrt{\frac{2E_s}{N_o}} \sin\left(\frac{\pi}{M}\right)\right] \cong \text{erfc}\left(\sqrt{\frac{E_s}{N_o}} \sin\left(\frac{\pi}{M}\right)\right) \cong \text{erfc}\left(\sqrt{(\log_2 M) \frac{E_b}{N_o}} \sin\left(\frac{\pi}{M}\right)\right), M \geq 4$$

where : $k = \log_2 M$, bits / symbol

2. For MPSK with gray coding, the bit error probability is: $P_{be,MPSK} = P_{se,MPSK} / k$
 3. The probability of bit error (BER) for **BPSK** ($M = 2$; $k = 1$):

$$P_{be,BPSK} = Q\left(\sqrt{\frac{2E_s}{N_o}}\right) = Q\left(\sqrt{\frac{2E_b}{N_o}}\right) = \frac{1}{2} \text{erfc}\left(\sqrt{\frac{E_b}{N_o}}\right)$$

4. Probability of symbol error rate for **QPSK** is: $P_{se,QPSK} = 2Q\left(\sqrt{\frac{E_s}{N_o}}\right) = \text{erfc}\left(\sqrt{\frac{E_s}{2N_o}}\right)$

5. Assuming no more than 1 bit error for each symbol error, the probability of bit error (BER) for **QPSK** ($M = 4$; $k = 2$) is half of its symbol error rate:

$$P_{be,QPSK} = Q\left(\sqrt{\frac{E_s}{N_o}}\right) = \frac{1}{2} \text{erfc}\left(\sqrt{\frac{E_s}{2N_o}}\right) = \frac{1}{2} \text{erfc}\left(\sqrt{\frac{E_b \log_2(4)}{2N_o}}\right) = \frac{1}{2} \text{erfc}\left(\sqrt{\frac{E_b}{N_o}}\right)$$

6. BPSK and QPSK have the same bit error probability (BER) because QPSK is configured as two BPSK signals modulating orthogonal components of the carrier. Offset QPSK has the same BER also.

Probability of Bit Error (BER) for M_{ary} PSK

In an Additive White Gaussian Noise (AWGN) channel

- The average probability of bit error (BER) for Multi-Level Phase Shift Keying using coherent detection can be approximated by:

$$P_{be,MPSK} \cong \frac{2}{\log_2 M} Q \left[\sqrt{\frac{2E_s}{N_o}} \sin\left(\frac{\pi}{M}\right) \right] \cong \frac{1}{\log_2 M} \operatorname{erfc} \left(\sqrt{(\log_2 M) \frac{E_b}{N_o}} \sin\left(\frac{\pi}{M}\right) \right), M \geq 4$$

M	k	Probability of Bit Error (BER): P_{be}	M	k	Probability of Bit Error (BER): P_{be}
4	2	$P_{be,QPSK} = \frac{1}{2} \operatorname{erfc} \left(\sqrt{\frac{E_b}{N_o}} \right)$	8	3	$P_{be,8PSK} \cong \frac{1}{3} \operatorname{erfc} \left(0.383 \sqrt{\frac{3E_b}{N_o}} \right)$
16	4	$P_{be,16PSK} \cong \frac{1}{4} \operatorname{erfc} \left(0.195 \sqrt{\frac{4E_b}{N_o}} \right)$	32	5	$P_{be,32PSK} \cong \frac{1}{5} \operatorname{erfc} \left(0.098 \sqrt{\frac{5E_b}{N_o}} \right)$
64	6	$P_{be,64PSK} \cong \frac{1}{6} \operatorname{erfc} \left(0.049 \sqrt{\frac{6E_b}{N_o}} \right)$	128	7	$P_{be,128PSK} \cong \frac{1}{7} \operatorname{erfc} \left(0.0245 \sqrt{\frac{7E_b}{N_o}} \right)$
256	8	$P_{be,256PSK} \cong \frac{1}{8} \operatorname{erfc} \left(0.0123 \sqrt{\frac{8E_b}{N_o}} \right)$	512	9	$P_{be,512PSK} \cong \frac{1}{9} \operatorname{erfc} \left(0.00613 \sqrt{\frac{9E_b}{N_o}} \right)$

Probability of Bit Error (BER): M_{ary} PSK

In an Additive White Gaussian Noise (AWGN) channel

Average Probability of Symbol Error for coherently detected Multi-Level PSK :

$$P_{se,MPSK} \cong 2Q \left[\sqrt{\frac{2E_s}{N_o}} \sin\left(\frac{\pi}{M}\right) \right] \cong \text{erfc} \left(\sqrt{\frac{E_s}{N_o}} \sin\left(\frac{\pi}{M}\right) \right) \cong \text{erfc} \left(\sqrt{(\log_2 M) \frac{E_b}{N_o}} \sin\left(\frac{\pi}{M}\right) \right), \text{ for } M \geq 4$$

where : $k = \log_2 M$, bits / symbol

Probability of Bit Error (BER) for coherent M-PSK : $P_{be,MPSK} = P_{se,MPSK} / k$, for $M > 4$

k, bits/ symbol =	1	2	3	4	5	
M signal levels =	2	4	8	16	32	
E_b/N_o , dB	E_b/N_o	Probability of Bit Error :		$P_{be,MPSK}$		
0	1.000	0.0786	0.0786	0.1162	0.1453	0.1513
2	1.585	0.0375	0.0375	0.0793	0.1218	0.1393
4	2.512	0.0125	0.0125	0.0458	0.0955	0.1247
6	3.981	0.0024	0.0024	0.0205	0.0677	0.1073
7	5.012	7.73E-04	7.73E-04	0.0120	0.0542	0.0975
8	6.310	1.91E-04	1.91E-04	0.0062	0.0414	0.0872
9	7.943	3.36E-05	3.36E-05	0.0027	0.0300	0.0765
10	10.000	3.87E-06	3.87E-06	0.0010	0.0202	0.0654
11	12.589	2.61E-07	2.61E-07	2.94E-04	0.0126	0.0543
12	15.849	9.01E-09	9.01E-09	6.34E-05	0.0070	0.0434
13	19.953	1.33E-10	1.33E-10	9.42E-06	0.0034	0.0332
14	25.119	6.81E-13	6.81E-13	8.76E-07	0.0014	0.0241
15	31.623	8.88E-16	8.88E-16	4.52E-08	4.79E-04	0.0163
16	39.811	0.0E+00	0.0E+00	1.11E-09	1.25E-04	0.0101

Probability of bit error (BER) for BPSK (M = 2) and QPSK (M = 4):

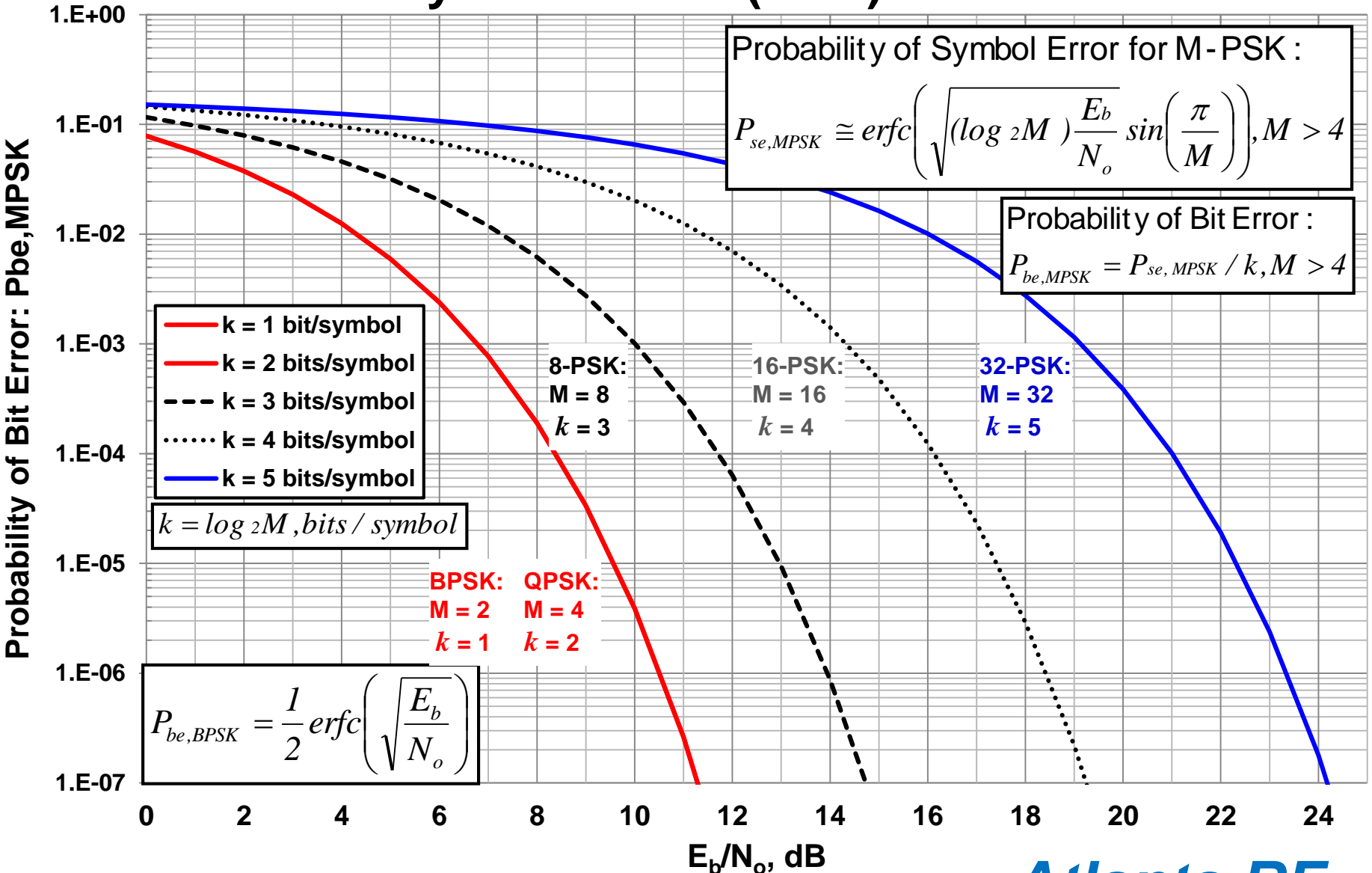
$$P_{be,BPSK} = P_{be,QPSK} = \frac{1}{2} \text{erfc} \left(\sqrt{\frac{E_b}{N_o}} \right)$$

The complementary error function: 'erfc' is built into most spreadsheet software programs, like: Excel.

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Probability of Bit Error (BER): Coherent M-PSK



E_b/N_o = Signal energy per bit over Noise density per bit

QAM: M-ary Quadrature Amplitude Modulation

Overview

1. Quadrature Amplitude Modulation is a method of amplitude modulation that allows two different signals sent simultaneously on the same carrier frequency, effectively doubling the bandwidth that can be transmitted.
2. By modulating two carriers at exactly the same frequency, but shifted by 90° , both the amplitude (ASK) and phase (PSK) of the carrier is modulated: QAM.
3. No binary QAM methods ($M = 2^1 = 2$), but lots of higher-order QAM:
 - A. 4QAM ($M = 2^2 = 4$ signal levels; 2 bits per symbol, like QPSK)
 - B. 16QAM ($M = 2^4 = 16$ signal levels; 4 bits per symbol)
 - C. 64QAM ($M = 2^6 = 64$ signal levels; 6 bits per symbol)
 - D. 256QAM ($M = 2^8 = 256$ signal levels; 8 bits per symbol) etc.
4. Features of Multi-Level Quadrature Amplitude Modulation (M_{ary} QAM):
 - A. Better BER Performance for higher 'M' states than equivalent M -PSK.
 - B. Bandwidth Efficient : Allows Power-Bandwidth Tradeoffs.
 - C. Requires Linear/Linearized Power Amplifiers.
 - D. Used in DVB ETSI Standard (European Telecommunications Standards Institute).
3. Used in applications where spectral efficiency is critical, e.g. DSL and high data rate wireless. Since both amplitude and phase are used to carry symbol information, M_{ary} QAM is very bandwidth efficient but requires a lot of transmit power for operation. See BER versus $E_b N_o$ graph.

QAM: M-ary Quadrature Amplitude Modulation

Basis of Operation

1. Quadrature Amplitude Modulation (QAM) is a 2-dimensional modulation:
 - A. Encodes baseband signal's data in carrier signal's phase and amplitude (both).
 - B. Combines amplitude & phase shift keying modulation: ASK & PSK, such that each signal corresponds to multiple bits.
 - C. $M = 2^k$ discrete signal levels. k bits coded/mapped into one symbol.
2. The general form of an M-ary QAM signal can be defined as:

$$s_i(t) = \sqrt{\frac{2E_{\min}}{T_s}} a_i \cos(2\pi f_c t) + \sqrt{\frac{2E_{\min}}{T_s}} b_i \sin(2\pi f_c t), \quad 0 < t < T_s, \quad i = 1, 2, 3, \dots, M$$

where E_{\min} is the energy of the signal with the lowest amplitude, and a_i and b_i are a pair of independent integers. Each carrier transmits half the symbol.

3. M-ary QAM does not have constant energy per symbol, nor constant distance between possible symbol states. E_s is not constant.... requires a linear channel. Power Spectrum & Bandwidth Efficiency of **QAM = MPSK**.
4. QAM finds extensive use in digital microwave radio links.

Probability of Bit Error (BER): M-QAM

In an Additive White Gaussian Noise (AWGN) channel

1. The average probability of bit error (BER) for Multi-Level QAM with square phase-state constellations using coherent detection can be approximated by:

$$P_{be, MQAM} \cong 4 \left(\frac{\sqrt{M} - 1}{\sqrt{M} \log_2 M} \right) Q \left(\sqrt{\frac{2E_{min}}{N_o}} \right) \cong 4 \left(\frac{\sqrt{M} - 1}{\sqrt{M} \log_2 M} \right) Q \sqrt{\frac{3E_{avg}}{(M-1)N_o}} = 2 \left(\frac{\sqrt{M} - 1}{\sqrt{M} \log_2 M} \right) \text{erfc} \left(\sqrt{\frac{3 \log_2 M E_b}{2(M-1)N_o}} \right)$$

2. The probability of bit error (BER) for popular M-QAM modulations are:

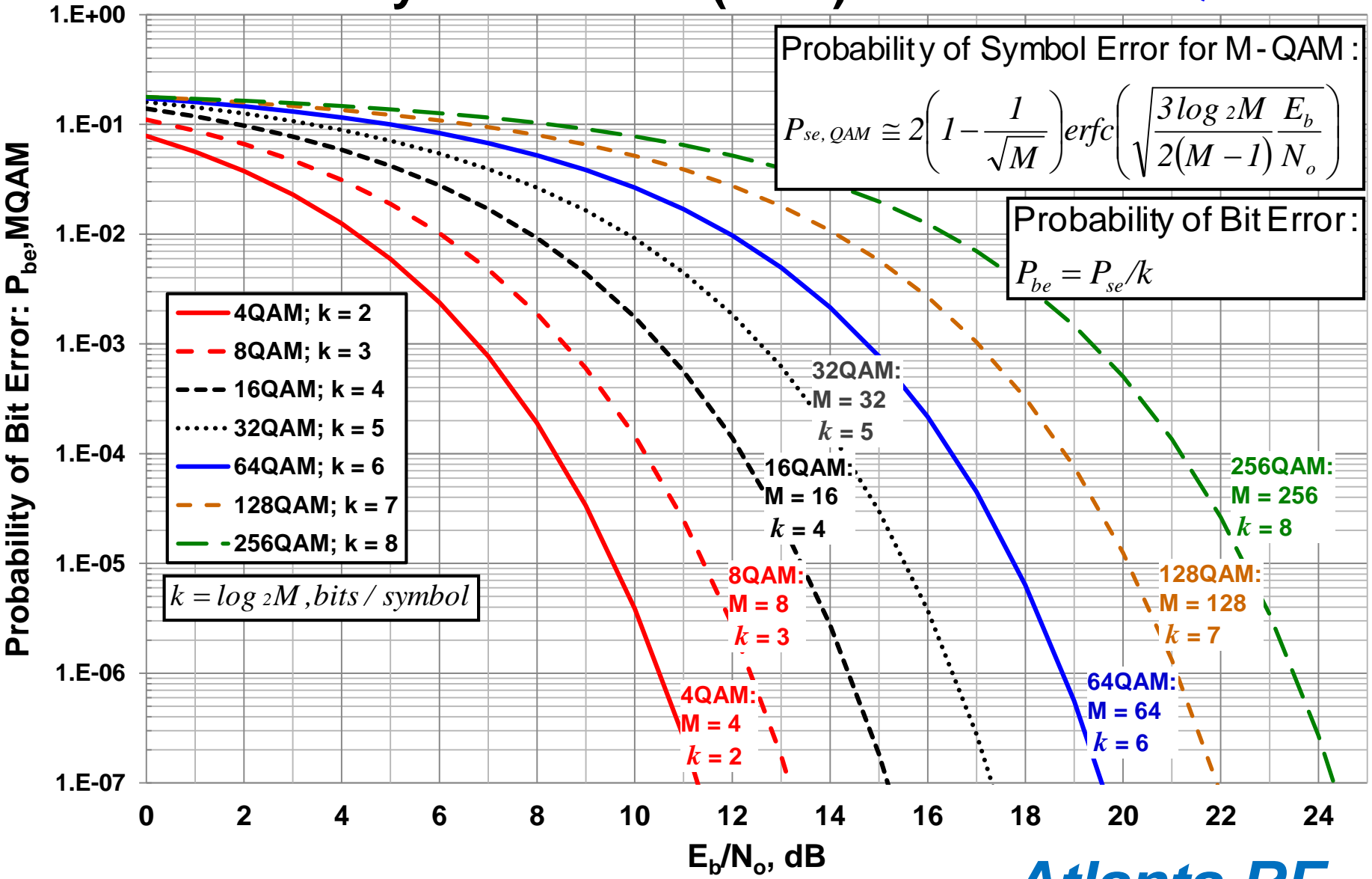
M	k	Probability of Bit Error (BER): P _{be}	M	k	Probability of Bit Error (BER): P _{be}
4	2	$P_{be,4QAM} \cong \frac{1}{2} \text{erfc} \left(\sqrt{\frac{E_b}{N_o}} \right)$	8	3	$P_{be,8QAM} \cong 0.43 \text{erfc} \left(\sqrt{\frac{9E_b}{14N_o}} \right)$
16	4	$P_{be,16QAM} \cong \frac{3}{8} \text{erfc} \left(\sqrt{\frac{2E_b}{5N_o}} \right)$	32	5	$P_{be,32QAM} \cong 0.33 \text{erfc} \left(\sqrt{\frac{15E_b}{62N_o}} \right)$
64	6	$P_{be,64QAM} \cong \frac{7}{24} \text{erfc} \left(\sqrt{\frac{E_b}{7N_o}} \right)$	128	7	$P_{be,128QAM} \cong 0.26 \text{erfc} \left(\sqrt{\frac{21E_b}{254N_o}} \right)$
256	8	$P_{be,256QAM} \cong \frac{15}{64} \text{erfc} \left(\sqrt{\frac{4E_b}{85N_o}} \right)$	512	9	$P_{be,512QAM} \cong 0.212 \text{erfc} \left(\sqrt{\frac{27E_b}{1022N_o}} \right)$

Probability of Bit Error (BER): M_{ary} QAM

In an Additive White Gaussian Noise (AWGN) channel

		4QAM	8QAM	16QAM	32QAM	64QAM	128QAM	256QAM	512QAM	1024QAM
k, bits/ symbol =		2	3	4	5	6	7	8	9	10
M signal levels =		4	8	16	32	64	128	256	512	1024
E_b/N_o , dB	E_b/N_o	Probability of Bit Error:				$P_{be, MQAM}$				
0	1.0000	0.0786	0.1107	0.1392	0.1603	0.1730	0.1782	0.1779	0.1738	0.1674
2	1.5849	0.0375	0.0661	0.0976	0.1255	0.1461	0.1585	0.1639	0.1640	0.1607
4	2.5119	0.0125	0.0312	0.0586	0.0890	0.1158	0.1352	0.1469	0.1520	0.1523
6	3.9811	0.0024	0.0102	0.0279	0.0544	0.0835	0.1087	0.1267	0.1373	0.1419
7	5.0119	7.73E-04	0.0048	0.0170	0.0393	0.0675	0.0945	0.1154	0.1289	0.1359
8	6.3096	1.91E-04	0.0019	0.0092	0.0265	0.0523	0.0800	0.1033	0.1197	0.1292
9	7.9433	3.36E-05	6.01E-04	0.0044	0.0164	0.0385	0.0656	0.0908	0.1098	0.1219
10	10.0000	3.87E-06	1.45E-04	0.0018	0.0092	0.0265	0.0517	0.0778	0.0993	0.1140
11	12.5893	2.61E-07	2.47E-05	5.65E-04	0.0045	0.0169	0.0388	0.0648	0.0881	0.1053
12	15.8489	9.01E-09	2.74E-06	1.39E-04	0.0019	0.0097	0.0275	0.0520	0.0765	0.0960
13	19.9526	1.33E-10	1.76E-07	2.42E-05	6.22E-04	0.0049	0.0181	0.0400	0.0647	0.0861
14	25.1189	6.81E-13	5.71E-09	2.76E-06	1.61E-04	0.0022	0.0108	0.0291	0.0530	0.0757
15	31.6228	8.88E-16	7.82E-11	1.84E-07	3.02E-05	7.72E-04	0.0058	0.0198	0.0417	0.0650
16	39.8107	0.0E+00	3.62E-13	6.25E-09	3.75E-06	2.17E-04	0.0027	0.0124	0.0312	0.0542
17	50.1187	0.0E+00	4.31E-16	9.07E-11	2.78E-07	4.50E-05	1.04E-03	0.0070	0.0220	0.0437
18	63.0957	0.0E+00	0.0E+00	4.52E-13	1.08E-08	6.35E-06	3.22E-04	0.0035	0.0144	0.0337
19	79.4328	0.0E+00	0.0E+00	5.83E-16	1.86E-10	5.54E-07	7.55E-05	0.0015	0.0086	0.0246
20	100.000	0.0E+00	0.0E+00	0.0E+00	1.15E-12	2.63E-08	1.24E-05	5.05E-04	0.0046	0.0168
21	125.893	0.0E+00	0.0E+00	0.0E+00	1.97E-15	5.85E-10	1.32E-06	1.35E-04	0.0021	0.0106
22	158.489	0.0E+00	0.0E+00	0.0E+00	0.0E+00	4.97E-12	7.99E-08	2.63E-05	8.08E-04	0.0060
23	199.526	0.0E+00	0.0E+00	0.0E+00	0.0E+00	1.27E-14	2.41E-09	3.44E-06	2.48E-04	0.0030
24	251.189	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	3.02E-11	2.72E-07	5.72E-05	0.0013
25	316.228	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	1.25E-13	1.14E-08	9.26E-06	4.50E-04

Probability of Bit Error (BER): Coherent M-QAM



E_b/N_o = Signal energy per bit over Noise density per bit

QAM in Digital Cable Services

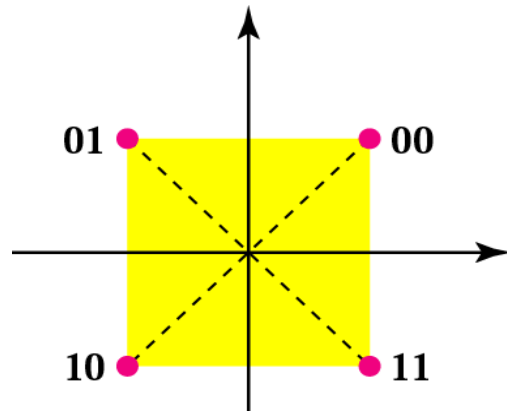
1. QAM is the standard for DOCSIS (Data Over Cable Service Interface Specification) and DVB-C (Digital Video Broadcasting – Cable).
2. Improves spectral efficiency, thereby providing more channels within a limited bandwidth:
 - A. 64 QAM can transmit 27Mbps or the equivalent of 6 to 10 analog channels or 1 HDTV signal over one 6MHz bandwidth.
 - B. 256 QAM can transmit 38.8 Mbps or the equivalent of 11 to 20 analog channels or 2 HDTV signals over one 6MHz bandwidth.
 1. An Standard Definition (SD) signal requires 2 to 3.5Mbps (depending on quality) and an High Definition (HD) signal requires 19.2 Mbps.
 2. New compression techniques can provide up to 3 HD signals on a 256 QAM carrier.
3. Assume an 8 bit sampler at 10kHz (voice): Bit rate is 80Kbps:

Modulation Type	BITS PER SYMBOL	UNIQUE SYMBOLS	SYMBOL RATE (BIT RATE = 80KBPS)
BPSK	1	2	80K Symbols/s
4 QAM (QPSK)	2	4	40K Symbols/s
16 QAM	4	16	20K Symbols/s
32 QAM	5	32	16K Symbols/s
64 QAM	6	64	13.3K Symbols/s
256 QAM	8	256	10K Symbols/s

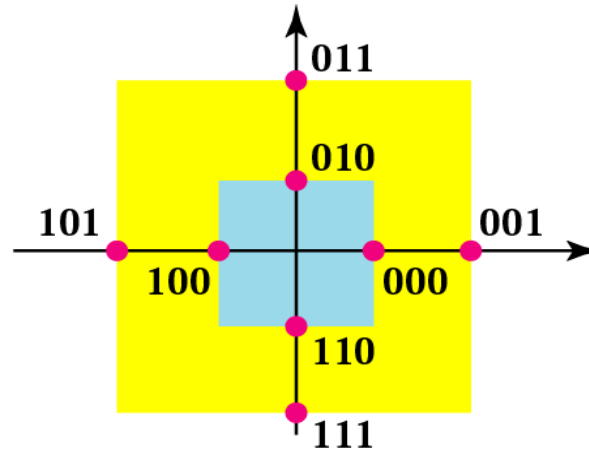
4-Level QAM and 8-Level QAM

Overview

1. For 4-QAM ($M = 2^2 = 4$ signal states) and 8-QAM ($M = 2^3 = 8$ signal states), the number of amplitude shifts is fewer than the number of phase shifts. Because amplitude changes are susceptible to noise and require greater 'shift differences' than do phase changes, the number of phase shifts used by a QAM system is always larger than the number of amplitude shifts.
2. Signal-space Constellation Diagrams for 4-QAM and 8-QAM:



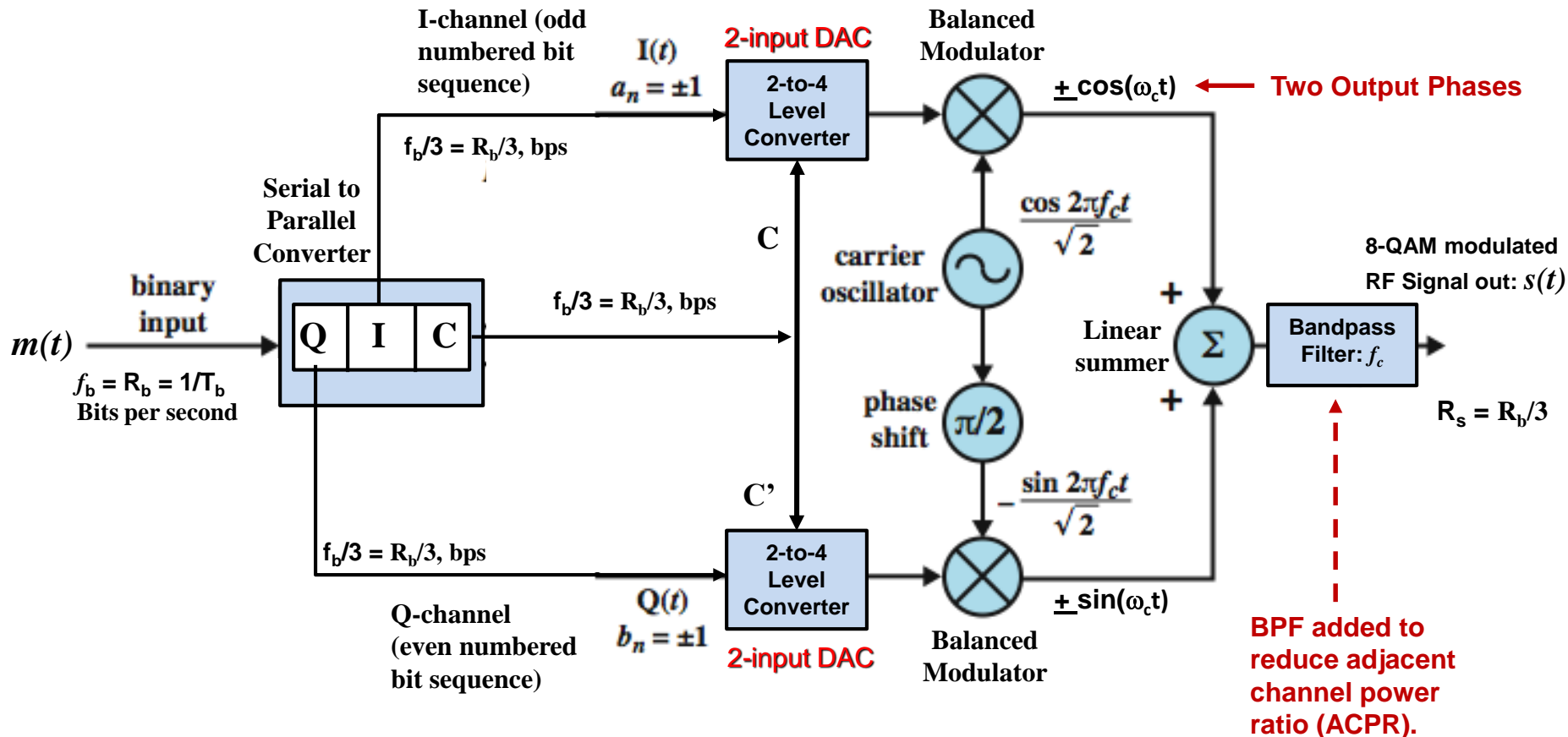
4-QAM
1 amplitude, 4 phases



8-QAM
2 amplitudes, 4 phases

Transmit Modulator

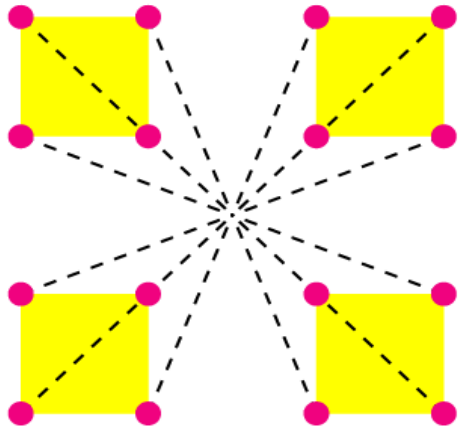
For: 8-Level Quadrature Amplitude Modulation (8-QAM)



- I, Q determines polarity; Binary '0' = Negative sign ("-").
Binary '1' = Positive sign ("+").
- C, \bar{C} determines the voltage level; Binary '1' = 1.307volts,
Binary '0' = 0.541volts.
- 2 levels + 2 polarities give 4 conditions in each (I & Q) channel.

16-Level QAM Constellation Diagrams: 16-QAM

3 amplitudes, 12 phases

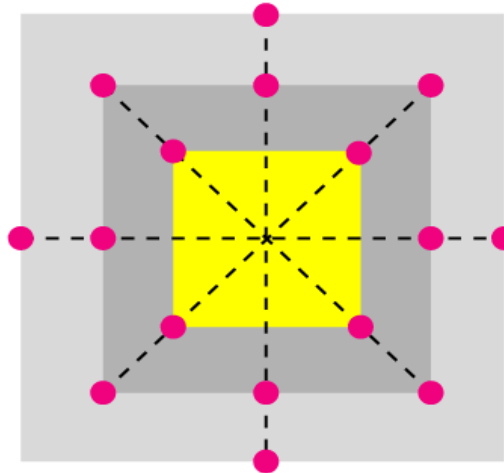


16-QAM

3 Amplitudes/12 Phases:

Handles noise best because of a greater ratio of phase shift to amplitude shift. ITU-T recommended.

4 amplitudes, 8 phases

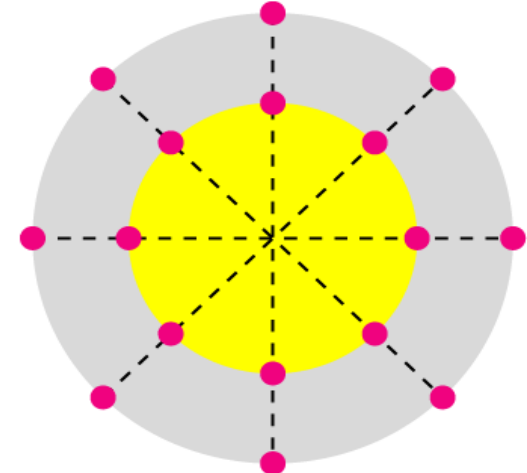


16-QAM

4 Amplitudes/8 Phases:

Measurable differences between signal shifts are increased and greater signal readability is ensured. ISO recommended.

2 amplitudes, 8 phases



16-QAM

2 Amplitudes/8 Phases:

Another signal-space representation for 16-QAM: Star 16-QAM.

ITU-T: International Telecommunications Standard.
ISO: International Organization of Standardization.

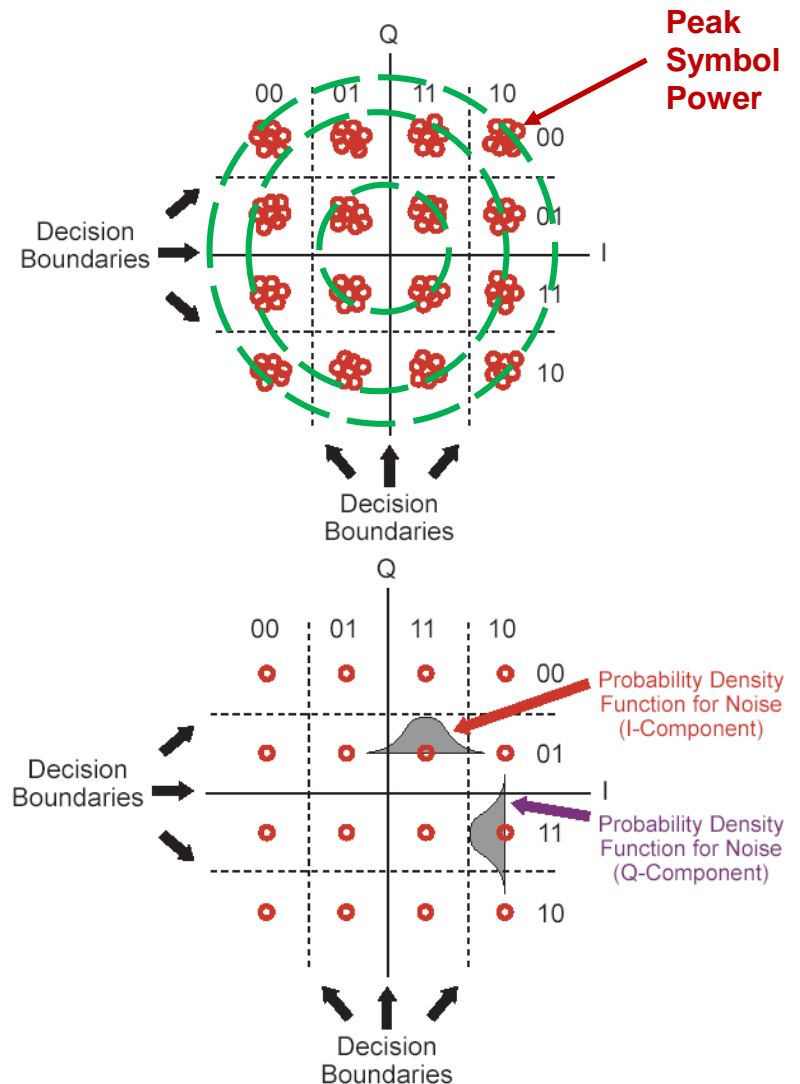
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Decision Boundaries for rectangular 16-QAM

Signal-space Constellation Diagram

Symbol Transmitted	Carrier Phase	Carrier Amplitude
0000	225°	0.33
0001	255°	0.75
0010	195°	0.75
0011	225°	1.0
0100	135°	0.33
0101	105°	0.75
0110	165°	0.75
0111	135°	1.0
1000	315°	0.33
1001	285°	0.75
1010	345°	0.75
1011	315°	1.0
1100	45°	0.33
1101	75°	0.75
1110	15°	0.75
1111	45°	1.0

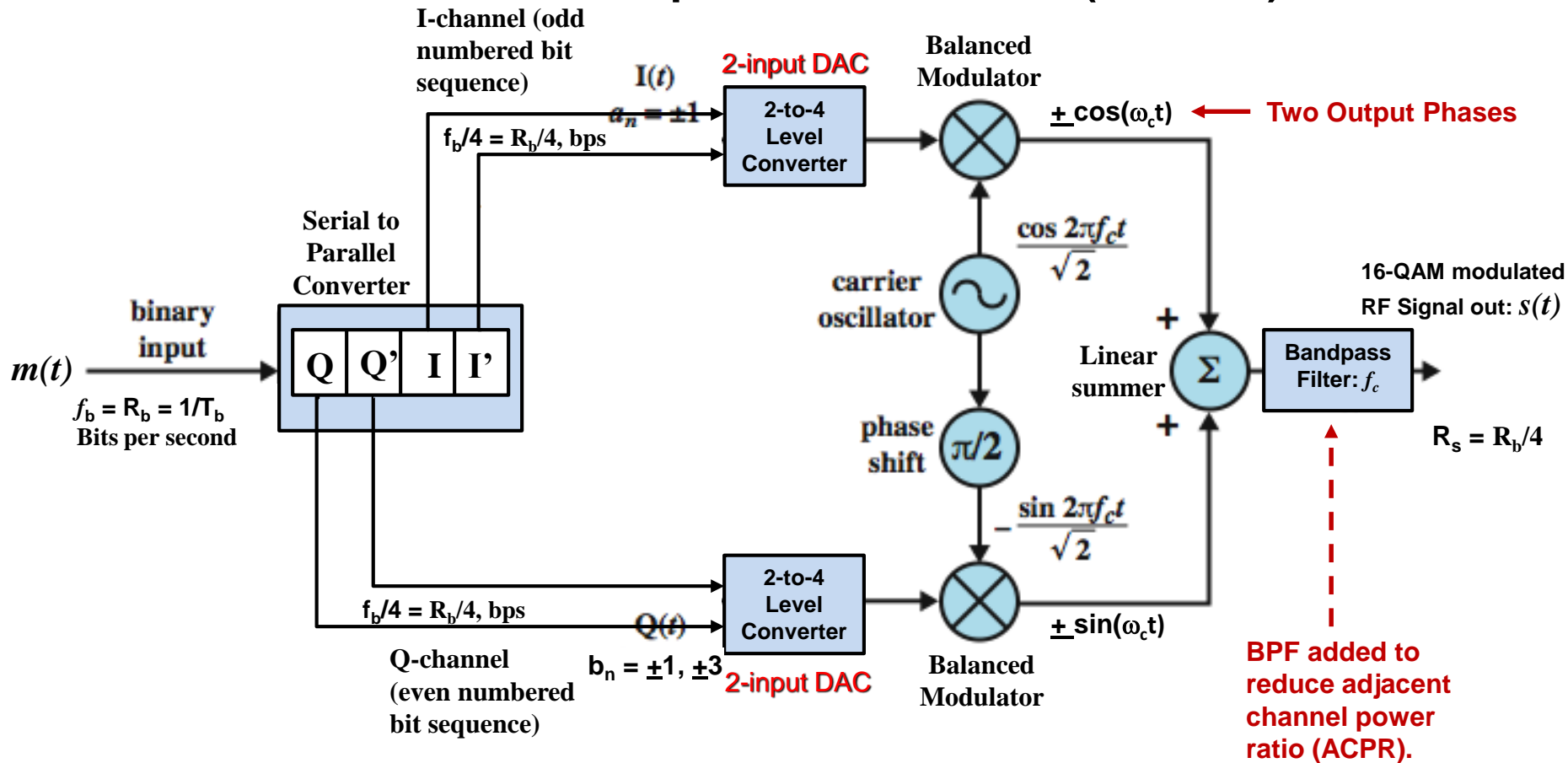


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Transmit Modulator

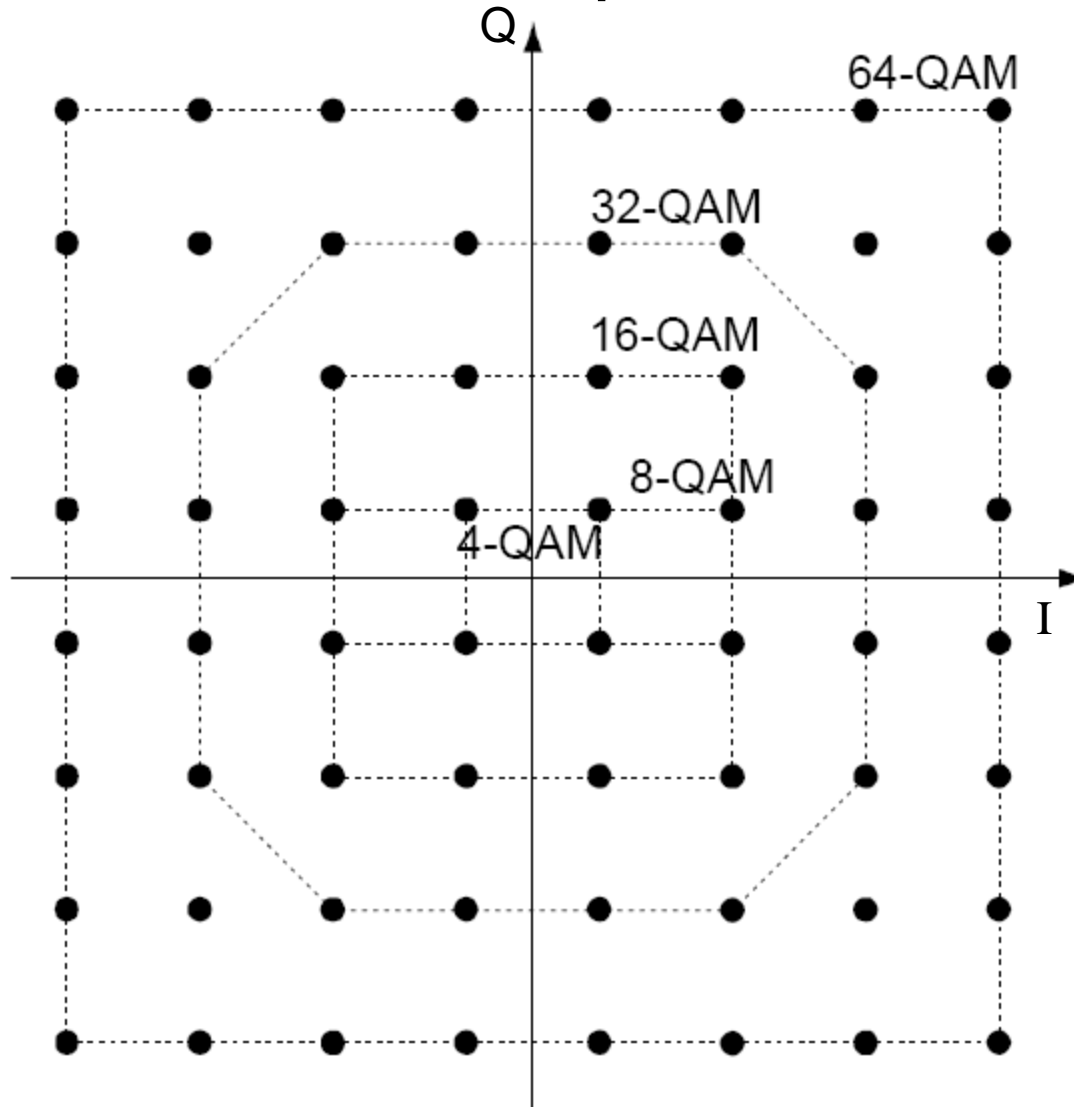
For: 16-Level Quadrature Amplitude Modulation (16-QAM)



- I, Q determine polarity; Binary '0' = Negative sign ("-").
Binary '1' = Positive sign ("+").
- I', Q' determines the voltage level/magnitude.

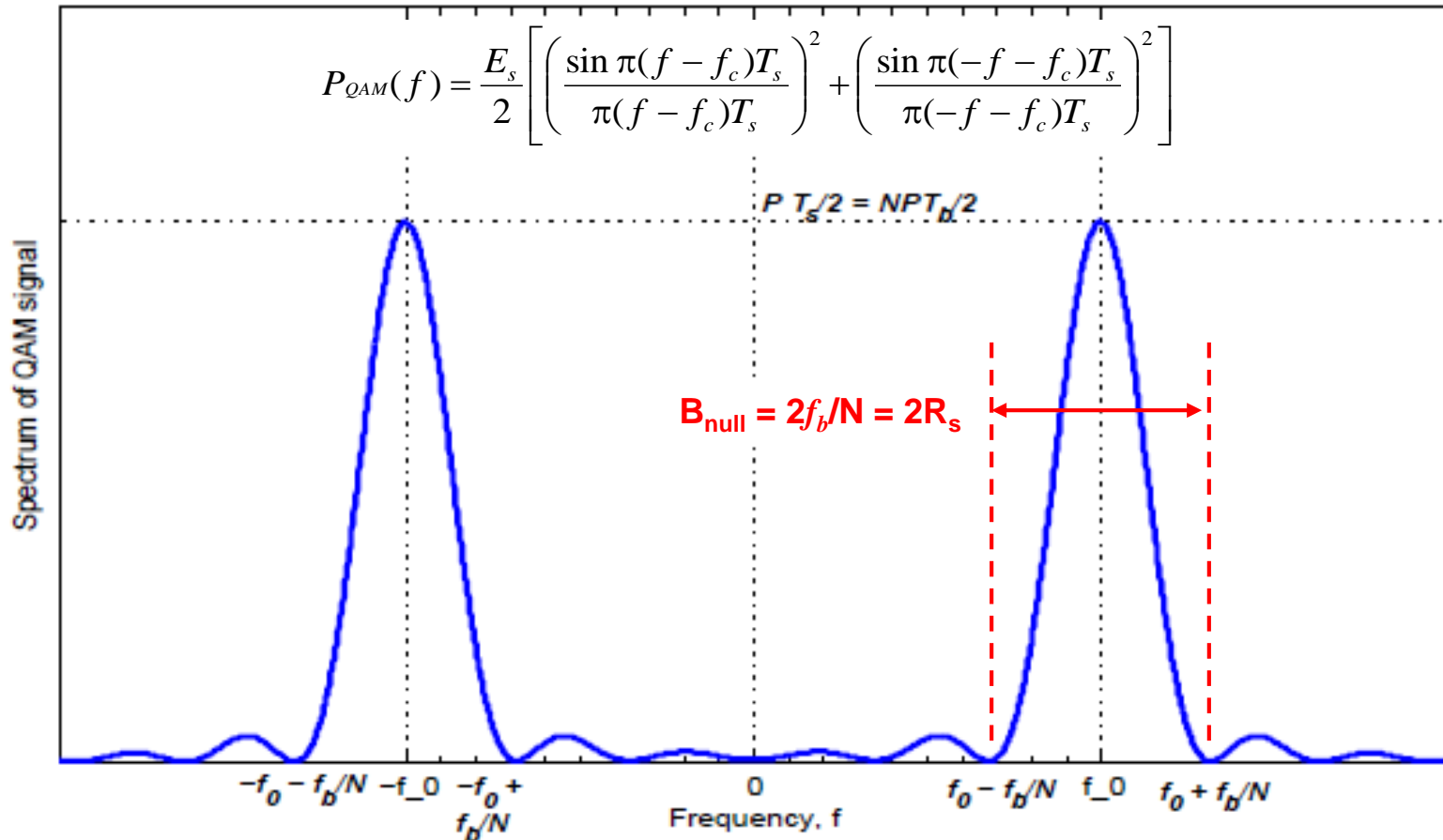
Signal-space representation: M-QAM

Various Multi-Level Quadrature Amplitude Modulation constellations



Bandpass Power Spectral Density: M-QAM signal

Assumes baseband rectangular pulses: 



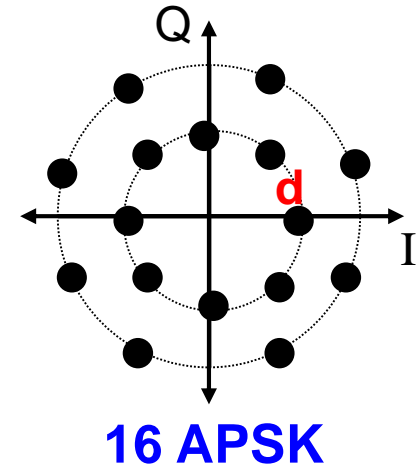
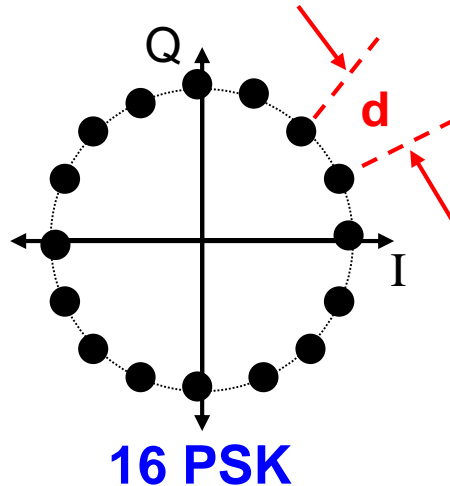
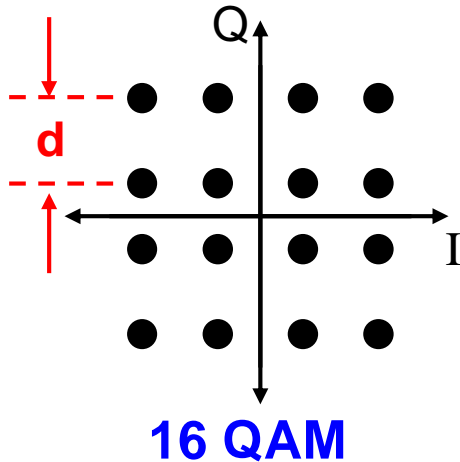
- Power Spectral Density of a M-QAM signal is the same as the PSD of a M-PSK signal.
- Null-to-Null RF transmission bandwidth: $B_{\text{null}} = (f_0 + f_b/N) - (f_0 - f_b/N) = 2f_b/N = 2R_s$.
- Spectral Efficiency: $\eta_{\text{BW}} = R_b/B_{\text{null}} = R_b/(2R_b/N) = (\log_2 M)/(1+\alpha)$, bits/second/Hertz.

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Multi-level (M-ary) Phase and Amplitude Modulation

As viewed from Signal-space Constellation Diagrams



1. Amplitude and phase shift keying can be combined to transmit several bits per symbol.
 - A. Often referred to as *linear* modulation, as they require linear amplification.
 - B. More bandwidth-efficient, but more susceptible to noise.
2. For 16-level modulation, 16QAM has the largest distance between constellation points, but requires very linear amplification. 16PSK has less stringent linearity requirements, but has less spacing between constellation points, and is therefore more affected by noise.

Comparison: M-PSK and M-QAM Modulations

Coherently detected M-PSK & M-QAM

Modulation Format	M Levels	k, bits per symbol	Spectral Efficiency	Null-to-Null Bandwidth	Carrier-to-Noise Ratio: Eb/No				
					BER 10 ⁻⁴	BER 10 ⁻⁶	BER 10 ⁻⁸	BER 10 ⁻¹⁰	BER 10 ⁻¹²
BPSK	2	1	0.5	2R _b	8.4dB	10.5dB	12dB	13dB	14dB
QPSK	4	2	1.0	R _b	8.4dB	10.5dB	12dB	13dB	14dB
8-PSK	8	3	1.5	2R _b /3	11.7dB	14dB	15.4dB	16.5dB	17.4dB
16-PSK	16	4	2.0	R _b /2	16.1dB	18.4dB	20dB	21dB	22dB
32-PSK	32	5	2.5	2R _b /5	21dB	23.4dB	24.9dB	26dB	27dB
4-QAM	4	2	1.0	R _b	8.4dB	10.5dB	12dB	13dB	13.9dB
8-QAM	8	3	1.5	2R _b /3	10.2dB	12.3dB	13.8dB	14.9dB	15.8dB
16-QAM	16	4	2.0	R _b /2	12.2dB	14.4dB	15.9dB	17dB	17.8dB
32-QAM	32	5	2.5	2R _b /5	14.3dB	16.5dB	18dB	19.1dB	20dB
64-QAM	64	6	3.0	R _b /3	16.5dB	18.8dB	20.3dB	21.4dB	22.3dB
256-QAM	256	8	4.0	R _b /4	21.2dB	23.5dB	25dB	26.1dB	27dB

➤ Spectral Efficiency for M-PSK and M-QAM:

- $\eta_{BW} = R_b/B_{null} = k/2 = 0.5 \log_2 M$, bits/second/Hertz.

➤ Null-to-Null Bandwidth assumes rectangular pulses.

Practical Applications

Multi-Level PSK and Multi-Level QAM

1. BPSK: Binary Phase Shift Keying:
 - A. Deep space satellite telemetry
 - B. Cable modems & RFID's.
 - C. WLAN IEEE 802.11b: 1 Mega bits/second.
2. QPSK: Quadrature Phase Shift Keying:
 - A. WLAN IEEE 802.11b: 2 M bps, 5.5 M bps, 11 M bps.
 - B. TETRA: Terrestrial Trunk Radio for emergency services/public safety.
 - C. Satellite & aircraft communications.
 - D. Digital Video Broadcasting: DVB-Sx, DVB-T with OFDM.
3. QAM: Quadrature Amplitude Modulation:
 - A. Telephone modems using 16-level QAM.
 - B. Terrestrial microwave links & DVB-Terrestrial.
 - C. Downstream of cable modem: 64QAM, 256QAM.
 - D. WLAN IEEE 802.11a/g:
 - 1) 16QAM for 24M bps, 36M bps;
 - 2) 64QAM for 38M bps and 54M bps.
 - E. High Speed Packet Access: HSDPA & HSUPA for LTE Cellular Systems.

Modulation Error Ratio (MER)

Useful metric for digital broadcast systems

1. Modulation Error Ratio (MER) in digital systems is analogous to Signal-to-Noise (S/N) or Carrier-to-Noise (C/N) used in analog systems.
2. Determining the MER of a digital signal is a critical part of determining how much margin the system has before failure. MER is a measure of how tightly symbols are recorded with respect to desired symbol location.
3. $MER(dB) = 20 \times \log(\text{RMS error magnitude/average symbol magnitude})$
4. Analog TV systems that have a poor C/N show up as a “snowy” picture.
5. A poor MER is not noticeable on the TV picture right up to the point of system failure.....the “Cliff Effect”.
6. As such, the TV cannot be used as a piece of test equipment anymore.
7. A 64-QAM set top converter requires better than 23 dB MER to operate.
8. A 256-QAM set top converter requires better than 28 dB MER to operate.
9. A 1024-QAM set top converter requires better than 33 dB MER to operate.
10. To allow for degradation, a margin (or headroom) of at least 3 to 4dB is preferred.

Summary: Digital Modulation, Part 3

Multi-Level PSK (M-PSK) and Multi-Level QAM (M-QAM)

1. Digital Modulation continues to dominate the world of data & voice communication with high throughput within a congested frequency spectrum at affordable cost.
2. Phase Shift Keying (PSK) and Quadrature Amplitude Modulation (QAM) are two-dimensional linear modulation techniques, with PSK providing a constant amplitude envelope.
 - A. High level M-ary schemes (such as 64-QAM) are very bandwidth efficient, but more susceptible to noise and require linear amplification.
3. Multi-Level PSK and Multi-Level QAM increases bandwidth efficiency at the cost of lower energy efficiency. Both modulation techniques are widely used in digital communication systems.
4. Look for additional presentations from *Atlanta RF* on Digital Modulation techniques, and visit our website: www.AtlantaRF.com to download these and other topics on Link Budget Analysis.

Refer to background material in Atlanta RF's presentations titled:

1. 'Link Budget – Getting Started' and
2. 'Link Budget: Digital Modulation Part 1' (Overview & M-ASK)
3. 'Link Budget: Digital Modulation Part 2' (M-FSK)

which can be downloaded from our website: www.AtlantaRF.com.

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Atlanta RF LLC was founded to provide engineering solutions, design software solutions, and product development solutions to the high-frequency RF/microwave industry in the areas of: Telecommunications (ground segment), Satellite (space segment) and military/defense (RF front-ends).

Through teamwork, Atlanta RF applies our diverse technical experience to your project's challenges with creative and innovative solutions while holding ourselves accountable for the results. With professionalism and commitment to our clients, Atlanta RF will be there for you, both today and tomorrow.

Contact Atlanta RF by e-mail at:

- Atlanta RF **Services** : Services@AtlantaRF.com
- Atlanta RF **Software** : Sales@AtlantaRF.com
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Or, contact Atlanta RF by phone at: 678-445-5544, to reach our Atlanta-area office in Georgia, USA, and discuss our support to your current or future projects & products.

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