RF Filters: An Overview

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Presentation Content: RF Filters

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2. Application of RF Filters:
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   B. Specific Applications.
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   B. Equi-Ripple (Chebyshev).
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Ideal Filters versus Actual Filters

A. Ideal Filter: A linear 2-port network that provides *perfect* transmission of signals for frequencies in a certain passband region, *infinite* attenuation for frequencies in the stopband region, and a *linear phase* response in the filter’s passband region (to reduce signal distortion).

B. Goal: The goal of an actual filter’s design is to approximate an ideal filter’s performance requirements, within acceptable tolerance, using real circuit components: Resistors: R, inductors: L, capacitors: C, transmission line sections, unit elements and resonating structures.

Magnitude plot for four ideal filters, along with their circuit symbols:
Importance of RF Filters

1. Frequency spectrum allocation and frequency spectrum preservation. RF signals consist of:
   A. Desired signals at desired frequencies.
   B. Undesired signals & undesired ‘noise’ at unwanted frequencies.
2. Signal interference reduction or elimination: Receiver protection.
3. Elimination of unwanted harmonics & intermodulation products generated from nonlinear devices, like:
   A. Frequency multipliers,
   B. Frequency mixers,
   C. Power amplifiers.
4. Signal processing & spectrum shaping.
5. Frequency multiplexing.
General Application of RF Filters

1. **Receivers**: Filters the incoming signal right after reception to remove external ‘white noise’ and external undesirable & interfering signals, thereby avoiding nonlinear operation of the Receiver’s Low Noise Amplifier and/or causing the Receiver to detect & process an unwanted signal.

2. **Transmitters**: Filters suppress much of the transmitter-generated harmonic frequencies, wide-band noise, intermodulation distortion (IMD) products, and out-of-band conversion frequencies.

3. **Communication systems**: The various frequency channels are very close, thus requiring bandpass filters with very narrow bandwidth & high out-of-band skirt attenuation/rejection.

4. **Diplexers**: Filters which provide high isolation (loss) between transmit signal frequencies and receive signal frequencies.

5. **Multiplexers**: Filters which separate or combine signal frequencies to different signal channels: Frequency channelization.

6. In detector circuits, frequency mixers and frequency multiplier applications, filters are used to attenuate/block unwanted high frequency intermodulation products.
Specific Applications of RF Filters

1. **Communication Systems:**
   A. Terrestrial Microwave Links: Receiver protection Filters, Transmitter Filters, Channel-dropping Filters, Transmitter harmonic frequency rejection Filters, Local Oscillator Filters, Mixer’s image-frequency rejection Filters.
   B. Satellite Systems:
      1) Spacecraft/Satellite: Front-end Receive Filters, Input multiplexer channelization Filters, Output multiplexer channelization Filters, Transmitter harmonic frequency rejection Filters.
      2) Earth Stations: LNA’s transmit reject Filters, High Power Amplifier’s harmonic frequency reject Filters, Up Converter & Down Converter Filters.

2. **Mobile and Cellular Systems:**
   A. Base Station Receive protection Filters.
   B. Base Station Transmitter Filters.
   C. Subscriber’s hand set Diplexer Filters.
   D. Satellite Mobile Applications:
      1) Aeronautical Transmit/Receive Systems.
      2) Maritime Satellite Terminals.
      3) Land Mobile Satellite Terminals.

3. **RADAR Systems:** Active & Passive.
4. **High RF Power Applications:** Various & many.
RF Filter Applications: Example
Transmit and Receive (Tx & Rx) Communication System

**Tx Bandpass Filter:**
Suppresses harmonic distortion from the Power Amplifier.

**Rx Bandpass Filter:**
- Suppresses the Power Amplifier’s harmonic distortion & limits the LNA input.
- Increases RF selectivity & suppresses LO leakages.

**Rx Bandpass Filter:**
- LO Bandpass Filter: Suppresses frequency harmonic RF signals generated inside Local Oscillator (LO).

**IF Bandpass Filter:**
- Supplies receive channel selectivity & suppresses mixer image frequency RF signals.
- Supplies receive channel selectivity & suppresses mixer image frequency RF signals.

**Radiated RF Signal**
- Antenna

**Power Amplifier**
- Duplexer

**TX RF Bandpass Filter**
- RX RF Bandpass Filter

**LNA (Low Noise Amplifier)**
- IF Mixer

**RX RF Filter**
- RX IF Filter

**To Low Frequency Signal Processing**

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RF Filter Classification

Electric Filters

Passive Filters
  - Lumped Components
    - LC
    - RC
    - With Resonators
      - SAWs
      - Crystal
      - Piezo-Electric
  - Distributed Components
    - Transmission Line
      - Interdigital
      - Comb Line
      - Coupled Line

Active Filters
  - LC Ladder
  - Blquads
  - Component Simulation
  - Dielectric Resonators
  - Coupled Resonators
Classification of RF Filters

Filters are classified according to the following parameters:

A. Frequency selection:
   1) Lowpass Filter: Low insertion loss below a specified cut-off frequency: $f_c$.
   2) Highpass Filter: Low insertion loss above a specified cut-off frequency: $f_c$.
   3) Bandpass Filter: Low insertion loss across a specified frequency band.
   4) Bandstop Filter: High insertion loss across a specified frequency band.

B. Amplitude response:
   1) Equi-ripple passband amplitude response: Chebyshev.
   3) Elliptic (Cauer), Bessel, Linear Phase, Gaussian, Pascal, etc.

C. Technology: Lumped, dielectric, planar, combline, waveguide, SAW, etc.

D. Frequency bandwidth: Narrow band (BW < 5%) or broad band (BW > 25%).

E. Reflection-type Filter or absorbing-type Filter:
   1) The majority of filters achieve out-of-band frequency attenuation by reflection, which results in high insertion loss to those signal frequencies and, also, results in very high Voltage Standing Wave Ration (VSWR).
   2) A small class of filters achieve out-of-band frequency attenuation by absorption of specified out-of-band signal frequencies.
## Technology & Application of RF Filters

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<tr>
<th>Frequency</th>
<th>UHF</th>
<th>L&amp; S-band</th>
<th>C-band</th>
<th>X- &amp; Ku-band</th>
<th>Ka-band</th>
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<td>Technology</td>
<td>Combline Dielectric Helical Planar SAW Waveguide</td>
<td>Combline Dielectric HTS Planar SAW Waveguide</td>
<td>Combline Dielectric HTS Planar Waveguide</td>
<td>Combline Dielectric Planar Waveguide</td>
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<td>Application</td>
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<td>PCS Satcom MMDS</td>
<td>Satcom</td>
<td>Satcom Link</td>
<td>LMDS Satcom</td>
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</table>

**Acronyms for ‘Technology’ of Filters:**
1. SAW: Surface Acoustic Wave.
2. HTS: High Temperature Superconductive.

**Acronyms for ‘Application’ of Filters:**
1. PCS: Personal Communication System.
2. MMDS: Multichannel Multipoint Distribution Service.
3. LMDS: Local Multipoint Distribution Service.
Common Transmission Media of RF Filters

Frequency Band Designation

As technology improves, the transmission media of filters can extend across broader frequencies.
## Features of Transmission Media for RF Filters

<table>
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<tr>
<th>Transmission Line Media (Construction)</th>
<th>Useful Frequency range (GHz)</th>
<th>Impedance Range (Ohms)</th>
<th>Cross-Sectional Dimensions</th>
<th>Quality Factor (Loss)</th>
<th>RF Power Rating</th>
<th>Active Device Mounting</th>
<th>Potential for Low-Cost Production</th>
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<td>100 to 500</td>
<td>Moderate to large</td>
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<td>High</td>
<td>Easy</td>
<td>Poor</td>
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<tr>
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<td>10 to 120</td>
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<td>Low</td>
<td>Low</td>
<td>Fair</td>
<td>Poor</td>
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<td>10 to 120</td>
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<td>Low</td>
<td>Low</td>
<td>Fair</td>
<td>Good</td>
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<td>Low</td>
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<td>Good</td>
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<td>20 to 150</td>
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<td>Moderate</td>
<td>Low</td>
<td>Easy</td>
<td>Fair</td>
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<td>Fair</td>
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<td>30 to 30</td>
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<td>Low</td>
<td>Poor</td>
<td>Good</td>
</tr>
<tr>
<td>Dielectric Line</td>
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<td>20 to 50</td>
<td>Moderate</td>
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<td>Low</td>
<td>Poor</td>
<td>Fair</td>
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### Technology

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<td>Coaxial cavity &amp; Comline</td>
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<td>Rectangular Waveguide</td>
<td>4000 to 15000</td>
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<tr>
<td>Dielectric Resonator</td>
<td>5000 to 50000</td>
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</table>
Illustration of planar TEM Bandpass Filters

1. Edge-coupled Band Pass Filter:
   A. Distributed transmission-line bandpass filter.
   B. Uses quarter wavelength long coupled lines.
   C. Prone to spurious amplitude response at higher frequencies.

2. Combinline Band Pass Filter:
   A. Physically compact bandpass filter.
   B. Uses quarter-wave transmission line resonators that are capacitively coupled.
   C. Good skirt rejection at higher frequencies.

3. Hairpin Band Pass Filter:
   A. Similar to the edge-coupled bandpass filter, but is considerably shorter.

4. Interdigital Band Pass Filter:
   A. Physically compact bandpass filter.
   B. Good skirt rejection at high frequencies.
   C. Short-circuited transmission lines that take the structure of interlaced fingers.
Illustration of some RF Bandpass Filters

8-pole Helical Filter

4-pole Dielectric-loaded Coaxial Bandpass Filter

4-pole Combline Bandpass Filter

3-pole Parallel Edge-Coupled Stripline Bandpass Filter

5-pole Hairpin Bandpass Filter
Illustration of some RF Lowpass Filters

Coaxial Lowpass Filter

Lumped Element Lowpass Filter

Suspended stripline band-pass filter consisting of low-pass filter (left side) and a high-pass filter (right side).

Distributed Microstrip Lowpass Filter
RF Filter Specifications
Factors that drive hardware design

1. Amplitude/magnitude response versus frequency characteristics:
   A. Frequency specifications: $f_c$, $f_0$ and bandwidth (BW = $f_{High} - f_{Low}$).
      1) For Lowpass & Highpass Filters: Cut-off frequency ($f_c$).
      2) For Bandpass & Bandstop Filters: Center frequency ($f_0$) & bandwidth.
   B. Passband insertion loss & passband amplitude ripple: $L_{ar}$; Return Loss.
      1) Passband frequency: Frequency range where filter passes all frequencies.
      2) Ripple: Difference between max and min of amplitude response in passband.
      3) Quality factor: Higher Q $\rightarrow$ lower passband insertion loss.
      4) Input & output impedances: $Z_{in}$ & $Z_{out}$.
   C. Out-of-band frequency (stopband) amplitude skirt rejection/attenuation.
   D. Spurious out-of-band frequency response, including higher-order modes.

2. Phase characteristics across passband frequency:
   A. Passband group delay variation: Affects Inter-symbol interference.

3. RF power handling requirement:
   A. Continuous Wave (CW) RF power & peak RF power.
   B. Multipactor effects & voltage breakdown.

4. Volume (size & shape) and mass/weight; In & out electrical connectors.

5. Environmental: Temperature, pressure, humidity, shock, vibration, etc.

6. Cost to manufacture and time to deliver.
Filters: Passband Amplitude Response

There are four main filter classes with passband amplitude response that approximate the ideal filter’s frequency response:

1. **Butterworth Filter**: Maximally-flat passband amplitude.
2. **Chebyshev Filter**: Equi-ripple passband amplitude response.
   - A. Inverted Chebyshev amplitude response: Chebyshev Type 2.
3. **Bessel Filter**: Maximally-flat time delay response.
4. **Elliptic-function (Cauer) amplitude response**.

Amplitude profiles vs Frequency of popular Lowpass Filters
Insertion Loss versus Frequency of several types of N = 7 Pole Lowpass Filters: $f_c = 1,000$ MHz

- **7PoleChebyshev**
- **7PoleButterworth**
- **7PoleBessel**
- **7PoleLinearPhase**
The Prototype Filter

A. A Prototype filter is an electronic filter design that serves as a template to produce a modified filter design for a particular application. The prototype filter is an example of a filter’s design from which the desired filter can be scaled or transformed.

B. Filters are required to operate at many different frequencies, bandwidths and impedances. The utility of a prototype filter comes from the property that all these other filters can be derived from it by applying a scaling factor to the components of the prototype filter. Thus, the filter design need only be carried out once in full, with other filters being obtained by simply applying a scaling factor.

C. Especially useful is the ability to transform from one frequency passband response to another: Lowpass response, Highpass response, Bandpass response and Bandstop response.

D. The prototype filter is most often expressed as a lowpass filter with a cutoff (or ‘corner’) frequency: $\omega_c = 1 \text{ radian/sec}$ and a characteristic impedance set to $R' = 1 \text{ Ohm}$. The desired filter is derived from this prototype lowpass filter by frequency scaling & impedance scaling.
Filter Synthesis: Insertion Loss Method

A. The insertion loss method enables a systematic way to design and synthesize a filter with various frequency responses.
B. The insertion loss method also enables a filter’s performance to be improved in a straightforward manner, at the expense of a ‘higher order’ or more complex filter.
C. A rational polynomial function: \(|H(\omega)|\) is used to approximate the ideal filter’s transfer function, but in amplitude only: \(A(\omega)\) or \(|S_{21}(\omega)|\).
D. Phase information is totally ignored when using the insertion loss method to synthesize the filter. Ignoring phase simplifies the actual synthesis method.
E. An L-C network is then derived that will produce this approximated amplitude response.
F. The attenuation: \(A(\omega)\) can be cast into a power attenuation ratio, called the Power Loss Ratio: \(P_{LR}\), which is related to \(A(\omega)^2\).
G. Modern filter synthesis can optimize a filter’s electrical circuit to meet both magnitude and phase requirements.
Filter Approximation Polynomials
Insertion Loss Method for Filter Synthesis

A. Every physically realizable filter circuit has a transfer function that is a rational polynomial in $s = \sigma + j\omega$.
B. We want to determine the classes of rational polynomials that approximate the “Ideal” lowpass filter response. Note: A Lowpass filter’s design can be used to derive a Highpass filter, a Bandpass filter or a Bandstop filter.
C. Four well known rational polynomial approximation functions that approach the ideal ‘brick wall’ filter’s response are:
Filter Classifications by Response Type
Insertion Loss Method for Filter Synthesis

Popular characteristic polynomials used to define the insertion loss of an N-section lowpass prototype filter are:

1. Butterworth or Maximally-flat passband amplitude response:
   \[ P_{LR} = 1 + \left( \frac{\omega}{\omega_o} \right)^{2n} \]

2. Chebyshev or equal-ripple passband amplitude response:
   \[ P_{LR} = 1 + \varepsilon^2 C_n^2(\omega/\omega_o), \text{ where } C_n^2 = \text{Chebyshev cosine polynomial}. \]

3. Inverse Chebyshev maximally-flat passband & equal-ripple stopband response:
   \[ P_{LR} = 1 + 1/ \left[ \varepsilon^2 C_n^2(\omega/\omega_o) \right]. \]

4. Elliptic function or Quasi-elliptic function response (equal-ripple in both pass band and stopband):
   \[ P_{LR} = 1 + \varepsilon^2 U_n^2(\omega/\omega_o). \]

5. Bessel-Thompson function response (Maximally-flat passband Phase/Group Delay).

**Insertion Loss** = \( 10\log_{10}(P_{LR}) \)
Prototype Low Pass Filter Ladder Circuits
Insertion Loss Method for Filter Synthesis

The prototype filter is a lowpass filter with a normalized cut-off frequency of $\omega' = 1$ radian/second and 1 ohm terminations at both the input/source: $g_0$ and output/load: $g_{n+1}$. Shown below are two N-section L-C ladder circuits for Lowpass filter prototypes and their electrical circuit element definitions:

\[ \begin{align*}
R_s &= g_0 = 1\Omega \\
L_2 &= g_2 \\
L_4 &= g_4 \\
C_1 &= g_1 \\
C_3 &= g_3 \\
C_5 &= g_5 \\
R_L &= g_{n+1}
\end{align*} \]

(a) Prototype Lowpass Filter with series inductor input: $L_1$.

(b) Prototype Lowpass Filter with shunt capacitor input: $C_1$.

where:
N = Order of the filter = Number of reactive elements in the filter.
g_0 = Generator’s source resistance or generator ‘s source conductance.
g_i = Inductance for series inductors or capacitance for shunt capacitors.
g_{n+1} = Load resistance if $g_N$ is a shunt capacitor or load conductance if $g_N$ is a series inductor.
When a Butterworth polynomial is used to define the insertion loss of an N-section lowpass filter, its power loss ratio is:

\[ P_{LR}(\text{Butterworth}) = 1 + \varepsilon^2 \left[ \left( \frac{\omega}{\omega_c} \right)^n \right]^2 \]

where:
- \( \varepsilon = 1 \) for a -3dB cutoff point.
- \( n \) = Order of the filter.
- \( \omega_c \) = Cutoff frequency, radians/sec.

Calculate the number of sections: \( n \) needed in a Butterworth filter as:

\[ n > \frac{\log_{10} \left( 10^{A/10} - 1 \right)}{2 \log_{10} \left( \omega_1 / \omega_c \right)} \]

where: ‘A’ is the attenuation in dB at some out-of-band frequency: \( \omega_1 \) ( \( \omega_1 > \omega_c \)).
Attenuation versus Normalized Frequency for an N-section Butterworth Filter

Normalized Frequency: \( \frac{\omega}{\omega_C} - 1 \)

Attenuation, dB

- \( N = 1 \)
- \( N = 2 \)
- \( N = 3 \)
- \( N = 4 \)
- \( N = 5 \)
- \( N = 6 \)
- \( N = 7 \)
- \( N = 8 \)
- \( N = 9 \)
- \( N = 10 \)
- \( N = 11 \)
- \( N = 12 \)
- \( N = 13 \)
- \( N = 14 \)
- \( N = 15 \)
Attenuation versus Normalized Frequency for an N-section Butterworth Filter

Normalized Cutoff Frequency: $f_c = 1$

Normalized Frequency: $\omega/\omega_c$
Lowpass Filter Prototype Circuit Element Values
Maximally-flat passband amplitude Butterworth Filter

Prototype Elements:

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<th>C₃</th>
<th>L₄</th>
<th>C₅</th>
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<td>1.41421</td>
<td>0.90798</td>
<td>0.31287</td>
</tr>
</tbody>
</table>

\[ g_i = \begin{cases} \text{Inductance for series inductors.} \\ \text{Capacitance for shunt capacitors.} \end{cases} \]

\[ g_i = 2 \sin \left( \frac{(2i - 1)\pi}{2n} \right) \]

\[ \omega_c' = 1 \text{ radian/sec} \]

\[ g_{n+1} = \begin{cases} \text{Load resistance if } g_n \text{ is a shunt capacitor.} \\ \text{Load conductance if } g_n \text{ is a series inductor.} \end{cases} \]

\[ g_0 = \begin{cases} \text{Source resistance.} \\ \text{Source conductance.} \end{cases} \]

Dual Circuits

Load resistance if \( g_n \) is a shunt capacitor. Load conductance if \( g_n \) is a series inductor.

Source resistance.
Source conductance.

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Lowpass Filter Prototype Circuit Element Values

Equi-ripple passband amplitude Chebyshev Filter

When a Chebyshev polynomial is used to define the insertion loss of an N-section lowpass prototype filter, its power loss ratio is:

\[ P_{LR}(\text{Chebyshev}) = 1 + \varepsilon^2 C_N^2 \left( \frac{\omega}{\omega_c} \right) \]

where:
- \( C_N \): Chebyshev polynomial of order \( N \).
- \( \omega_c \): Cutoff frequency, radians/sec.

\( P_{LR} \) oscillates between 1 & 1+\( \varepsilon^2 \).

Calculate the number of sections needed in a Chebyshev filter as:

\[ N > \cosh^{-1} \sqrt{\left( \frac{10^A/10 - 1}{10^{L_{\text{ar}}/10} - 1} \right)} \]

where: ‘\( A \)’ is the attenuation in dB at some out-of-band frequency: \( \omega_1 \) (\( \omega_1 > \omega_c \)).

Chebyshev lowpass prototype circuit element values:

\[ g_0 = 1; \quad g_1 = \frac{2a_1}{\gamma}; \quad g_k = \frac{4a_{i-1}a_i}{b_{i-1}g_{i-1}} \]

\[
g_{n+1} = \begin{cases} 
1 & \text{for } n \text{ odd} \\
\coth^2 \left( \frac{\beta}{4} \right) & \text{for } n \text{ even}
\end{cases}
\]

where:
- \( \beta = \ln(\coth \frac{L_{\text{ar}}}{17.34}) \); \( \gamma = \sinh(\frac{\beta}{2n}) \)
- \( a_i = \sin \left( \frac{(2i-1)\pi}{2N} \right) \); \( i = 1,2,...,N \)
- \( b_i = \gamma^2 + \sin^2 \left( \frac{i\pi}{N} \right) \); \( i = 1,2,...,N \)

\( N \): Order of the filter.

\( L_{\text{ar}} \): Maximum passband ripple, dB.

Feb-2015 www.AtlantaRF.com
Attenuation versus Normalized Frequency for an N-section Chebyshev Filter: 0.01dB passband ripple

Case of $L_{AR} = 0.01\text{dB}$
Passband Ripple;
Return Loss $> 26\text{ dB}$

Normalized Frequency: $[\omega/\omega_C - 1]$
Attenuation versus Normalized Frequency for an N-section Chebyshev Filter: 0.01dB passband ripple

Normalized Cutoff Frequency: $f_c = 1$

Case of $L_{AR} = 0.01\text{dB}$ Passband Ripple; Return Loss $> 26 \text{ dB}$

Normalized Frequency: $\omega/\omega_c$
Lowpass Prototype Circuit Element Values
0.01 dB equi-ripple passband amplitude Chebyshev Filter

Passband Ripple: $L_{ar} = 0.01$ dB; Passband Return Loss $> 26$dB

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$R_s = g_0 = 1$Ω

Dual Circuits

$g_{n+1} = \begin{cases} 
\text{Load resistance if } g_n \text{ is a shunt capacitor} \\
\text{Load conductance if } g_n \text{ is a series inductor}
\end{cases}$

$g_0 = \begin{cases} 
\text{Source resistance} \\
\text{Source conductance}
\end{cases}$
Attenuation versus Normalized Frequency for an N-section Chebyshev Filter: 0.04dB passband ripple

Case of $L_{AR} = 0.04$dB Passband Ripple; Return Loss > 20 dB

Normalized Frequency: $[\omega/\omega_C - 1]$
Attenuation versus Normalized Frequency for an N-section Chebyshev Filter: 0.04dB passband ripple

Normalized Cutoff Frequency: $f_c = 1$

Case of $L_{AR} = 0.04$dB
Passband Ripple; Return Loss > 20 dB
Lowpass Prototype Circuit Element Values

0.04 dB equi-ripple passband amplitude Chebyshev Filter

Passband Ripple: $L_{ar} = 0.04$ dB; Passband Return Loss $> 20$dB

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$L_1 = C_2 = L_3 = C_4 = L_5 = C_6 = L_7 = C_8 = L_9 = C_{10} = L_{11}$

$R_s = g_0 = 1\Omega$

$g_n + 1 = \begin{cases} 
\text{Load resistance if } g_n \text{ is a shunt capacitor} \\
\text{Load conductance if } g_n \text{ is a series inductor}
\end{cases}$

$\omega'_c = 1 \text{ radian/sec}$

Dual Circuits

$g_0 = \begin{cases} 
\text{Source resistance} \\
\text{Source conductance}
\end{cases}$
Attenuation versus Normalized Frequency for an N-section Chebyshev Filter: 0.1dB passband ripple

Case of $L_{AR} = 0.1\text{dB}$
Passband Ripple; Return Loss > 16 dB

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Attenuation versus Normalized Frequency for an N-section Chebyshev Filter: 0.1dB passband ripple

- Normalized Cutoff Frequency: \( f_c = 1 \)
- Case of \( L_{AR} = 0.1\text{dB} \)
- Passband Ripple: Return Loss > 16.4 dB

\[ \text{Normalized Frequency: } \omega/\omega_c \]
Lowpass Prototype Circuit Element Values
0.1 dB equi-ripple passband amplitude Chebyshev Filter

Passband Ripple: $L_{ar} = 0.1$ dB; Passband Return Loss > 16dB

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$R_s = g_0 = 1\Omega$

Dual Circuits

$g_{n+1} = \begin{cases} 
\text{Load resistance if } g_n \text{ is a shunt capacitor} \\
\text{Load conductance if } g_n \text{ is a series inductor.}
\end{cases}$

$g_0 = \begin{cases} 
\text{Source resistance.} \\
\text{Source conductance.}
\end{cases}$
Attenuation versus Normalized Frequency for an N-section Chebyshev Filter: 0.5dB passband ripple

Case of $L_{AR} = 0.5\text{dB}$
Passband Ripple; Return Loss > 9.6 dB
Attenuation versus Normalized Frequency for an N-section Chebyshev Filter: 0.5dB passband ripple

Normalized Cutoff Frequency: $f_c = 1$

Case of $L_{AR} = 0.5\text{dB}$ Passband Ripple; Return Loss $> 9.6 \text{ dB}$
Lowpass Prototype Circuit Element Values
0.5 dB equi-ripple passband amplitude Chebyshev Filter

Passband Ripple: $L_{ar} = 0.5$ dB; Passband Return Loss $> 9.6$dB

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$R_s = g_0 = 1\Omega$

$C_1 = g_1$
$C_2 = g_2$
$C_3 = g_3$
$C_4 = g_4$
$C_5 = g_5$
$C_6$
$C_7$
$C_8$
$C_9$
$C_{10}$

$R_L = g_{N+1}$

$\omega'_c = 1$ radian/sec

$g_{n+1} = \begin{cases} 
\text{Load resistance if } g_n \text{ is a shunt capacitor.} \\
\text{Load conductance if } g_n \text{ is a series inductor.}
\end{cases}$

$g_0 = \begin{cases} 
\text{Source resistance.} \\
\text{Source conductance.}
\end{cases}$
Attenuation versus Normalized Frequency for N-section Bessel Filters

Cutoff Frequency $f_c = 1,000$ MHz

Normalized Frequency: $\omega/\omega_c$
Lowpass Prototype Circuit Element Values
Maximally-flat Time Delay Bessel Filter

<table>
<thead>
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<th>Prototype Elements:</th>
<th>g₁</th>
<th>g₂</th>
<th>g₃</th>
<th>g₄</th>
<th>g₅</th>
<th>g₆</th>
<th>g₇</th>
<th>g₈</th>
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<th>L₁</th>
<th>C₂</th>
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<th>C₄</th>
<th>L₅</th>
<th>C₆</th>
<th>L₇</th>
<th>C₈</th>
<th>L₉</th>
<th>C₁₀</th>
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</table>

Rₛ = g₀ = 1Ω

ωᶜ' = 1 radian/sec

\( g_{n+1} = \left\{ \begin{array}{ll}
\text{Load resistance if } g_n \text{ is a shunt capacitor.} \\
\text{Load conductance if } g_n \text{ is a series inductor.}
\end{array} \right\}

source resistance.

Load conductance if \( g_n \) is a series inductor.
Lowpass Prototype Circuit Element Values
Linear Phase Filter with equi-ripple passband phase error of 0.05°

<table>
<thead>
<tr>
<th>Prototype Elements:</th>
<th>$g_1$</th>
<th>$g_2$</th>
<th>$g_3$</th>
<th>$g_4$</th>
<th>$g_5$</th>
<th>$g_6$</th>
<th>$g_7$</th>
<th>$g_8$</th>
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$R_s = g_0 = 1\Omega$

$C_1 = g_1$
$C_3 = g_3$
$C_5 = g_5$
$L_2 = g_2$
$L_4 = g_4$

$R_L = g_{N+1}$

$\omega'_c = 1$ radian/sec

$g_{n+1} = \begin{cases} 
\text{Load resistance if } g_n \text{ is a shunt capacitor.} \\
\text{Load conductance if } g_n \text{ is a series inductor.}
\end{cases}$

Dual Circuits

$g_0 = \begin{cases} 
\text{Source resistance.} \\
\text{Source conductance.}
\end{cases}$
Comparison of some Lowpass Prototype Circuit Element Values vs. Response Type

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<th>Filter's Response:</th>
<th>Prototype Elements:</th>
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<th>$g_6$</th>
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<td>Bessel Filter</td>
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<td>0.86903</td>
<td>1.10516</td>
<td>2.26590</td>
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</table>

For $R(\text{source}) = R(\text{load}) = g_0 = g_n + 1 = 1$ Ohm; $\omega_c = 1$ radian/second.
A Note about Passband Amplitude Ripple

Filters synthesized using the Insertion Loss Method produce a passband VSWR that is related to the passband’s amplitude ripple:

1. The passband’s amplitude ripple is directly related to the filter’s input/output VSWR as:

\[ \Gamma = \sqrt{1 + 10^{\frac{L_{ar}}{10}}} \]

\[ VSWR = \frac{1 + \Gamma}{1 - \Gamma} \]

As such, a higher passband amplitude ripple produces a higher passband VSWR, as shown in the table. Often, higher passband VSWR can adversely effect the performance in communication systems and, therefore, it seems ‘silly’ to publish any filter’s design whose ripple exceeds \( L_{ar} > 0.5 \text{dB} \) passband ripple.

<table>
<thead>
<tr>
<th>Passband Ripple, ( L_{ar}, \text{dB} )</th>
<th>Reflection Coefficient ( \Gamma )</th>
<th>Return Loss, ( \text{db} )</th>
<th>VSWR</th>
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<td>0.005</td>
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</table>

Butterworth
Frequency Scaling the Prototype Lowpass Filter

1. The basis for frequency normalization of filters is:
   A. A prototype filter’s response can be scaled (shifted) to a different frequency range by dividing the reactive elements: L & C, by a frequency scale factor.
   B. The frequency scale factor is the ratio of the desired filter’s reference frequency to the prototype filter’s reference frequency (often $\omega_c' = 1$ rad/sec):
   
   \[
   \text{Frequency Scale Factor} = \frac{\text{Desired Frequency}}{\text{Existing Frequency}}
   \]
   C. The cutoff frequency: $f_c$ is selected as the desired frequency for lowpass & highpass filters, while the center frequency: $f_0$ and fractional bandwidth is selected as the desired frequency factor for bandpass & bandstop filters.

2. Bandpass & Bandstop Filters: It is sometimes desirable to compute two geometrically related frequencies that correspond to a given bandwidth. When given the center frequency: $f_0$ and the bandwidth: BW, the lower cutoff frequency: $f_1$, and upper cutoff frequency: $f_2$ are computed as:

\[
f_1 = \sqrt{\left(\frac{\text{BW}}{2}\right)^2 + f_0^2 - \frac{\text{BW}}{2}}
\]
\[
f_2 = \sqrt{\left(\frac{\text{BW}}{2}\right)^2 + f_0^2 + \frac{\text{BW}}{2}}
\]
Frequency Mapping the Lowpass Prototype
The Insertion Loss Method for Filter Synthesis

Frequency mapping the lowpass prototype filter into the desired filter type:

Frequency mapping to a lowpass filter:

\[ \omega \leftrightarrow \frac{\omega}{\omega_c} \]

\( \omega_c = \text{Cutoff frequency} \)

Frequency mapping to a highpass filter:

\[ \omega \leftrightarrow -\left( \frac{\omega}{\omega_c} \right)^{-1} \]

Frequency mapping to a bandpass filter:

\[ \omega \leftrightarrow \frac{\omega_0}{BW} \left( \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right) \]

\( \omega_0 = \text{Center frequency}; \ BW = \text{Bandwidth} = \omega_2 - \omega_1 \)

Frequency mapping to a bandstop filter:

\[ \omega \leftrightarrow -\frac{BW}{\omega_0} \left( \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right)^{-1} \]

Electrical circuit components realized after frequency mapping:

<table>
<thead>
<tr>
<th>Series Circuit Components</th>
<th>Shunt Circuit Components</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Low-pass prototype</th>
<th>Low-pass</th>
<th>High-pass</th>
<th>Bandpass</th>
<th>Bandstop</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>L = g_k</strong></td>
<td>( \frac{L}{\omega_c} )</td>
<td>( \frac{1}{\omega_c L} )</td>
<td>( \frac{L}{BW} )</td>
<td>( \frac{1}{(BW)L} )</td>
</tr>
<tr>
<td><strong>C = g_k</strong></td>
<td>( \frac{C}{\omega_c} )</td>
<td>( \frac{1}{\omega_c C} )</td>
<td>( \frac{C}{BW} )</td>
<td>( \frac{1}{(BW)C} )</td>
</tr>
</tbody>
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<th>Electrical Circuit Components</th>
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</table>

- **Low-pass**
- **High-pass**
- **Bandpass**
- **Bandstop**
Impedance Scaling the Prototype Lowpass Filter

1. The basis for impedance normalization of filters is:
   A. A prototype filter’s impedance level can be scaled to a different value by adjusting the prototype element values using an impedance scale factor.
   B. The impedance scale factor is the ratio of the desired filter’s impedance level to the prototype filter’s impedance level (often: \( Z'_0 = 1 \text{ ohm} \)):

   \[
   \text{Impedance Scale Factor} = \frac{\text{Desired Impedance}}{\text{Prototype Impedance}}
   \]

   C. To impedance-transform the prototype filter:
      1) Multiply all resistances & inductances by the impedance scale factor.
      2) Divide all capacitors by the impedance scale factor.

   C. For many RF filters, the filter’s desired impedance is often: \( Z_0 = 50 \text{ ohms} \).

2. Since the filter is a linear circuit, we can multiply all impedances by some factor without changing the transfer function of the filter. This leaves the frequency response unchanged, but impedance-scales the desired filter to its required impedance level. Its frequency response remains as a lowpass, highpass, bandpass or bandstop filter.
Frequency Transformation and Impedance Scaling of Low Pass Prototype

1. Once the Lowpass Prototype Filter is designed, the cut-off frequency: $\omega_c$ can be transformed to other frequencies.
2. Furthermore, the Lowpass Prototype Filter can be frequency mapped into other filter types, such as: Highpass Filter, Bandpass Filter, and Bandstop Filter.
3. This frequency scaling and transformation entails changing the value and configuration of the electrical circuit elements of the Lowpass Prototype Filter.
4. Finally, the impedance presented by the filter can also be scaled, from unity to another impedance value, like: $Z_o = 50$ ohms.
Lowpass & Highpass Filter Transformations
Frequency-scaling & Impedance-scaling circuit components

Frequency & Impedance scaling to a **Lowpass Filter**:

1. The series inductor: \( g_k = L_k \) is transformed into a series inductor: \( L \) with a value:

\[
L' = \frac{Z_0 g_k}{\omega_c}
\]

2. The shunt capacitor: \( g_k = C_k \) is transformed into a shunt capacitor: \( C \) with a value:

\[
C' = \frac{g_k}{Z_0 \omega_c}
\]

\[\omega_c = 2\pi f_c = \text{Desired cut-off frequency.}\]
\[Z_0 = \text{Impedance of the system, Ohms.}\]

Frequency & Impedance scaling to a **Highpass Filter**:

1. The series inductor: \( g_k = L_k \), is transformed into a series capacitor: \( C \) with a value:

\[
C' = \frac{1}{Z_0 g_k \omega_c}
\]

2. The shunt capacitor: \( g_k = C_k \), is transformed into a shunt inductor: \( L \) with a value:

\[
L' = \frac{Z_0}{g_k \omega_c}
\]
Bandpass & Bandstop Filter Transformations
Frequency-scaling & Impedance-scaling circuit components

**Bandpass Filter:**

1. The series inductor: \( g_k = L_k \) is transformed into a series \( LC \) circuit with element values:
   \[
   L'_k = \frac{Z_0 g_k}{\Delta \omega_0}, \quad C'_k = \frac{\Delta}{\omega_0 Z_0 g_k}
   \]

2. The shunt capacitor: \( g_k = C_k \) is transformed into a shunt \( LC \) circuit with element values:
   \[
   C'_k = \frac{g_k}{\Delta Z_0 \omega_0}, \quad L'_k = \frac{\Delta Z_0}{\omega_0 g_k}
   \]

**Bandstop Filter:**

1. The series inductor: \( g_k = L_k \), is transformed into a parallel \( LC \) circuit with element values:
   \[
   L'_k = \frac{\Delta Z_0 g_k}{\omega_0}, \quad C'_k = \frac{1}{\omega_0 \Delta Z_0 g_k}
   \]

2. The shunt capacitor: \( g_k = C_k \), is transformed into a series \( LC \) circuit with element values:
   \[
   C'_k = \frac{\Delta g_k}{\omega_0 Z_0}, \quad L'_k = \frac{Z_0}{\omega_0 \Delta g_k}
   \]

**Center frequency:**
   \[
   \omega_o = \frac{\omega_1 + \omega_2}{2} \quad \text{or} \quad \sqrt{\omega_1 \omega_2}
   \]

**Fractional bandwidth:**
   \[
   \Delta = \frac{\omega_2 - \omega_1}{\omega_o}
   \]
Frequency-Scaling & Impedance-Scaling Circuit Components of Low Pass Prototype (LPP) Filter

LPP to Lowpass

\[ L \left( g_k \right) = \frac{Z_o L_k}{\omega_c} \]

LPP to Highpass

\[ \frac{I}{\omega_c L_k Z_o} \]

LPP to Bandpass

\[ \frac{L_k Z_o}{\omega_o \Delta} \quad \frac{\Delta}{\omega_o L_k Z_o} \]

LPP to Bandstop

\[ \frac{L_k \Delta Z_o}{\omega_o} \]

Series Component

Shunt Component

Center frequency:

\[ \omega_o = \frac{\omega_1 + \omega_2}{2} \quad \text{or} \quad \sqrt{\omega_1 \omega_2} \]

Fractional bandwidth:

\[ \Delta = \frac{\omega_2 - \omega_1}{\omega_o} \]

Note: The inductor always multiplies with \( Z_o \) while the capacitor divides with \( Z_o \).
Electrical Circuit Configuration
Lowpass Filter & Highpass Filter after Freq. & Impedance Scaling

![Diagram of Electrical Circuit Configuration]

**Lowpass filter** derived from the lowpass prototype filter.

**Highpass filter** derived from the lowpass prototype filter.
Electrical Circuit Configuration
Bandpass Filter & Bandstop Filter after Freq. & Impedance Scaling

\[
\begin{align*}
R & \quad L_N \quad C_N \quad L_5 \quad C_5 \quad L_3 \quad C_3 \quad L_1 \quad C_1 \quad R_L=1 \\
& \quad \cdots
\end{align*}
\]

**Bandpass filter** derived from the prototype lowpass filter.

\[
\begin{align*}
R & \quad L_N \quad C_N \quad L_5 \quad C_5 \quad L_4 \quad C_4 \quad L_2 \quad C_2 \quad L_1 \quad C_1 \quad R_L=1 \\
& \quad \cdots
\end{align*}
\]

**Bandstop filter** derived from the prototype lowpass filter.
Summary of Steps in Filter Design & Synthesis
The Insertion Loss Method for Filter Synthesis

Step 1: Filter Specifications
A. Filter Type: Lowpass or Highpass or Bandpass or Bandstop.
B. Frequency Response: Butterworth, Chebyshev or Bessel.
C. Desired frequency of operation: \( f_c \) or \( f_o + \) bandwidth.
D. Passband & stopband frequency range.
E. Maximum allowed attenuation (for Equal ripple).

Step 2: Low Pass Prototype Design
A. Minimum Insertion Loss level, Number of Filter Order/Elements by using insertion loss values.
B. Circuit configuration: Shunt capacitor model or Series inductance model.
C. Draw the low-pass prototype circuit ladder diagram.
D. Determine circuit elements’ values from the Lowpass Prototype Table.

Step 3: Scaling and Conversion
A. Determine if any modification to the prototype table is required (for high pass, band pass and band stop).
B. Frequency scale & impedance scale prototype lowpass circuit element values to obtain the desired filter’s real circuit element values. . . . then implement.
Example: Butterworth Lowpass Filter

1. Specifications:
   A. Maximally-flat lowpass filter; $f_c = 2.0$ GHz ; $Z_0 = 50$ Ohms.
   B. Out-of-band skirt attenuation > 15 dB at 3.0 GHz.

2. Filter Design Solution:
   A. Normalized frequency at 3 GHz: $(\omega/\omega_c - 1) = (3GHz/2GHz - 1) = 0.5$.
   B. Refer to graph titled: ‘Attenuation vs Normalized Frequency for N-section Butterworth filter’. Find best curve that is above the point located at Attenuation = 15dB when $(\omega/\omega_c - 1) = 0.5$. Result: $N = 5$. Alternately, you can calculate the number of circuit elements needed using:

   $$n > \frac{\log_{10}(10^{A/10} - 1)}{2\log_{10}(\omega_1 / \omega_c)}$$

   C. For $N = 5$ circuit elements, calculate prototype element values for Butterworth filter or use element values from Butterworth filter table: $g_0 = g_6 = 1.0$, $g_1 = 0.61803$ ; $g_2 = 1.61803$ ; $g_3 = 2.0$ ; $g_4 = 1.61803$ ; $g_5 = 0.61803$.
   D. The prototype lowpass ladder circuit is shown below:
Example: Butterworth Lowpass Filter (cont.)

3. Frequency-scale & impedance-scale lowpass prototype filter’s circuit elements to: \( f_c = 2 \text{ GHz} \) and \( Z_0 = 50 \text{ ohms} \):

\[
C_1 = \frac{g_1}{Z_0\omega_c} = \frac{0.618}{(50)(2\pi \times 2 \times 10^9)} = 0.984 \text{ pf}
\]

\[
L_2 = \frac{Z_0g_2}{\omega_c} = \frac{50 \times 1.618}{(2\pi \times 2 \times 10^9)} = 6.438 \text{ nh}
\]

\[
C_3 = \frac{g_3}{Z_0\omega_c} = \frac{2.00}{(50)(2\pi \times 2 \times 10^9)} = 3.183 \text{ pf}
\]

\[
L_4 = \frac{Z_0g_4}{\omega_c} = \frac{50 \times 1.618}{(2\pi \times 2 \times 10^9)} = 6.438 \text{ nh}
\]

\[
C_5 = \frac{g_5}{Z_0\omega_c} = \frac{0.618}{(50)(2\pi \times 2 \times 10^9)} = 0.984 \text{ pf}
\]

4. Implement the final (desired) Butterworth Lowpass Filter:

![Filter Circuit Diagram]
Atlanta RF LLC was founded to provide engineering solutions, design software solutions, and product development solutions to the high-frequency RF/microwave industry in the areas of: Telecommunications (ground segment), Satellite (space segment) and military/defense (RF front-ends).

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